Image formation through atmospheric turbulence
(& introduction to adaptive optics)

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[https://lagrange.oca.eu/carbillet/enseignement/M1-MAUCA/]
[http://mauca.unice.fr/index.php/documents-and-ressources-for-atmospheric-turbulence-image-formation-adaptive-optics/]

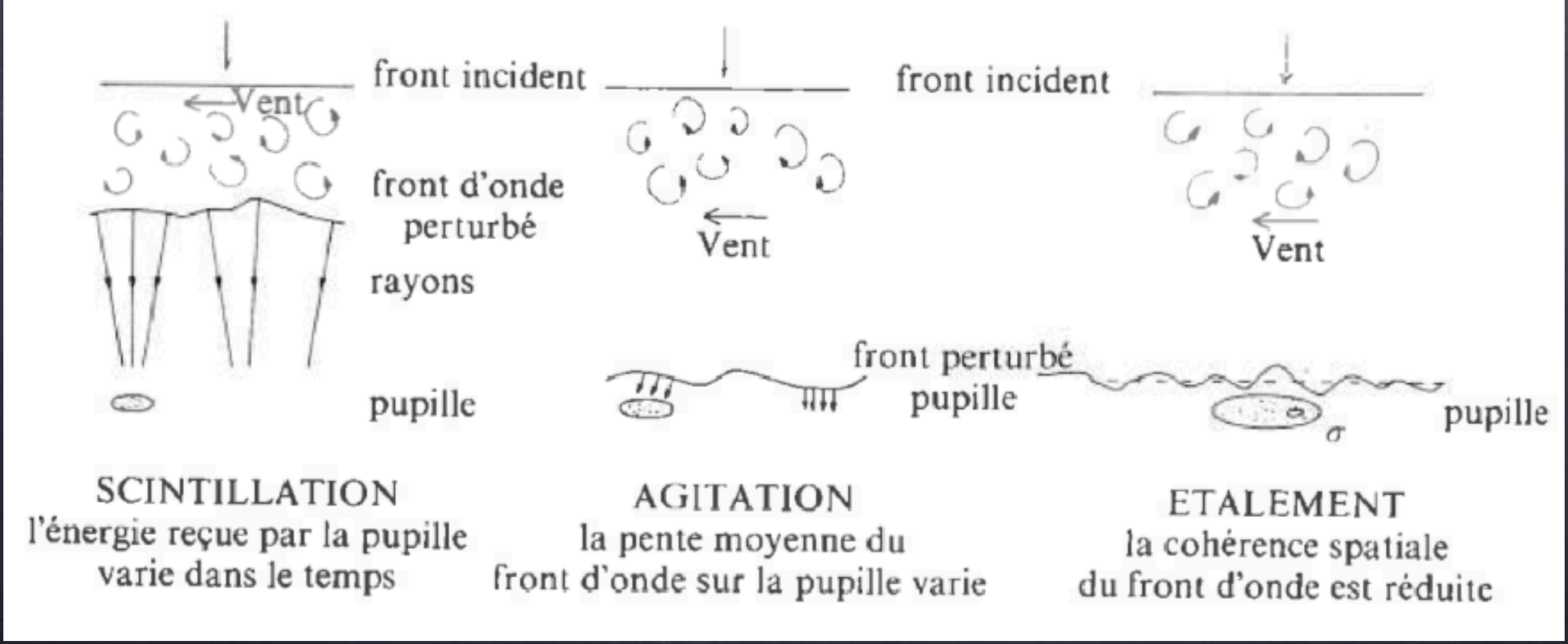
Menu

- High-angular resolution (HAR) imaging in astronomy
- Atmospheric turbulence (reminder)
- Numerical modelling of pertubed wavefronts
- Formation of resulting image (+detector noises!)
- Introduction to *Speckle Interferometry*
- Introduction to adaptive optics (AO)
- AO error budget & post-OA PSF morphology
- Anisoplanatic error study (ideal AO system)

Images & turbulence - 1

The image formed through turbulent atmosphere (optically speaking) is degraded:

- Scintillation (due to intensity fluctuation in the pupil).
- Agitation (due to angle-of-arrival variation).
- Spreading (due to a loss of spatial coherence).



(Illustration from: Pierre Léna, Astrophysique - Méthodes physiques de l'observation, CNRS Éd. (2me éd.), p.177)

Images & turbulence - 2

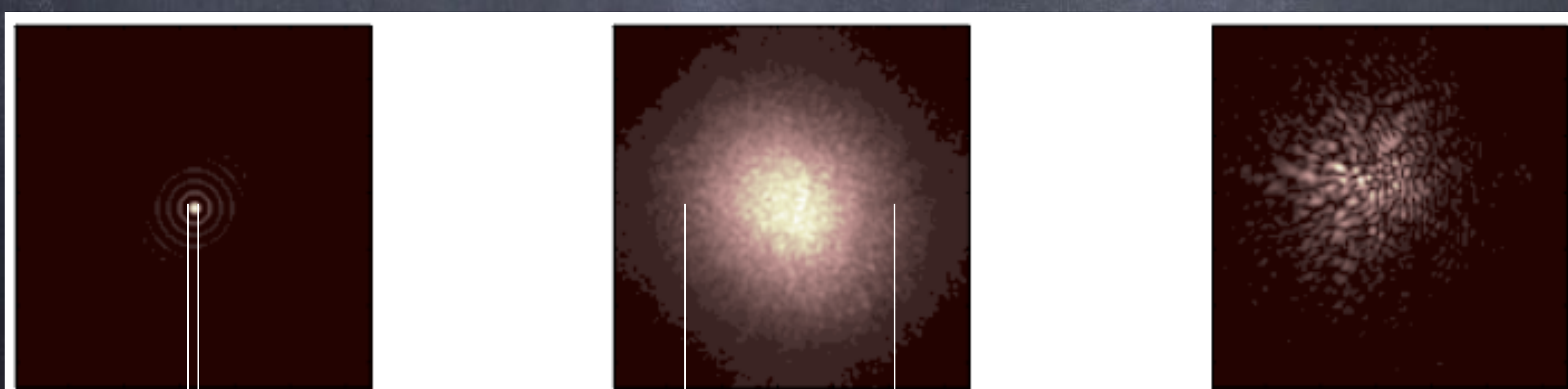
The object-image relation between the intensity $I(\alpha)$ in the image plane (i.e. the focal plane of the telescope) and the brightness $O(\alpha)$ of the object (in the sky) is a relation of convolution implying the point-spread function (PSF) $S(\alpha)$ of the whole ensemble telescope+atmosphere, with α the pointing direction:

$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

Images & turbulence - 3

$$I(\vec{\alpha}) = O(\vec{\alpha}) * S(\vec{\alpha})$$

This relation is valid notably at the condition that the system is invariant by translation (everything happens within the isoplanatic domain)...



l/D $\quad$ l/r_0

Images & turbulence - 4

Some orders of magnitude concerning the turbulent atmosphere:

	λ=500nm	λ=2.2mm
Fried parameter r_0	→ 10 cm	60 cm
velocity of the turbulent layers (v)	→ 10 m/s	id.
=> image FWHM ($\varepsilon \simeq \lambda / r_0$)	→ 1"	~1"
=> evolution time ($t_0 \propto r_0 / v$)	→ 3 ms	18 ms

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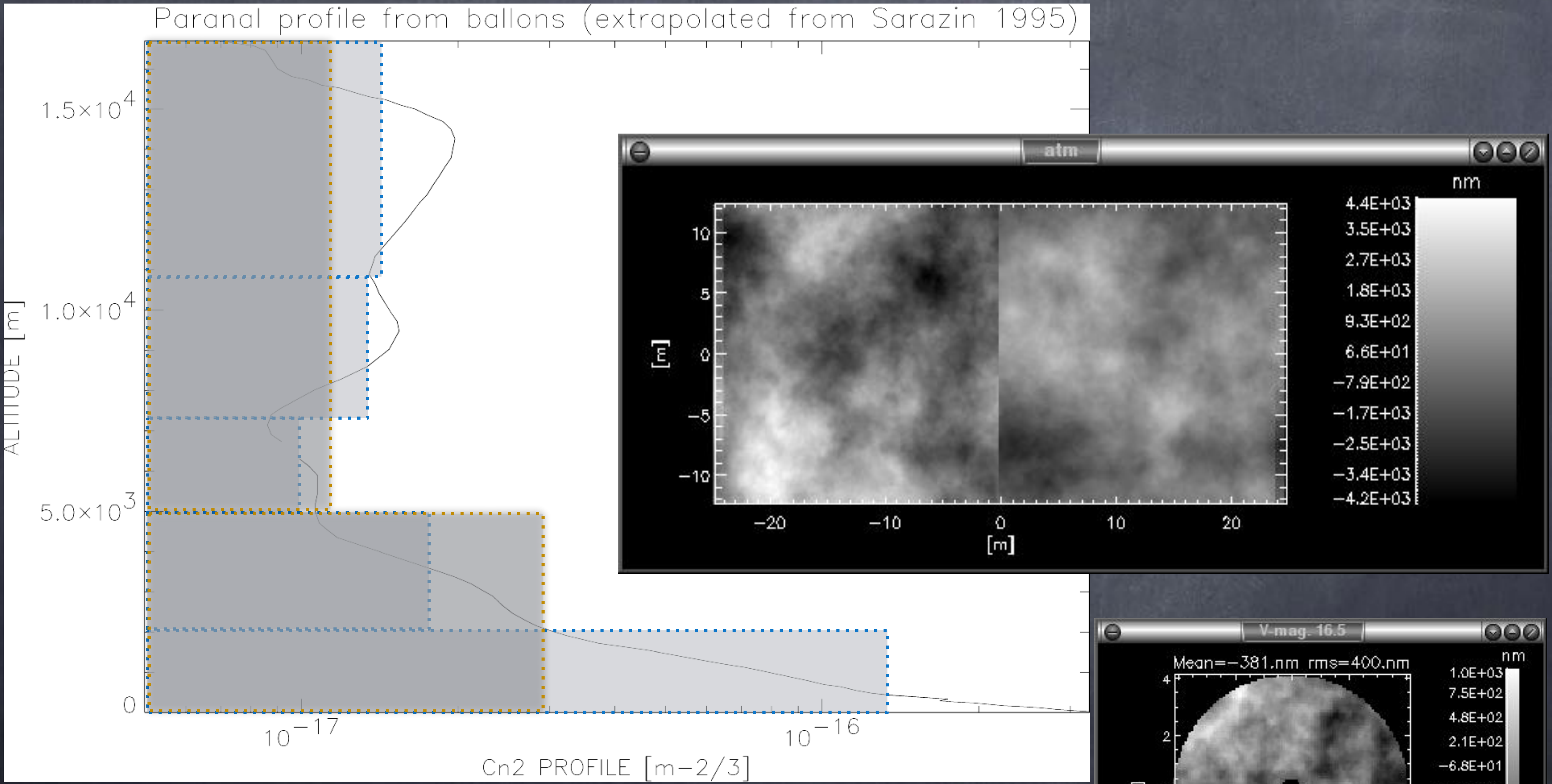

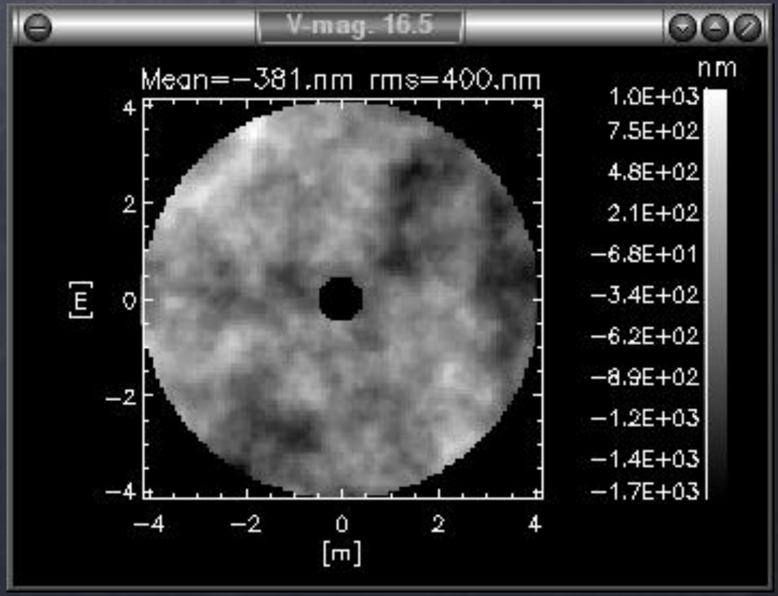

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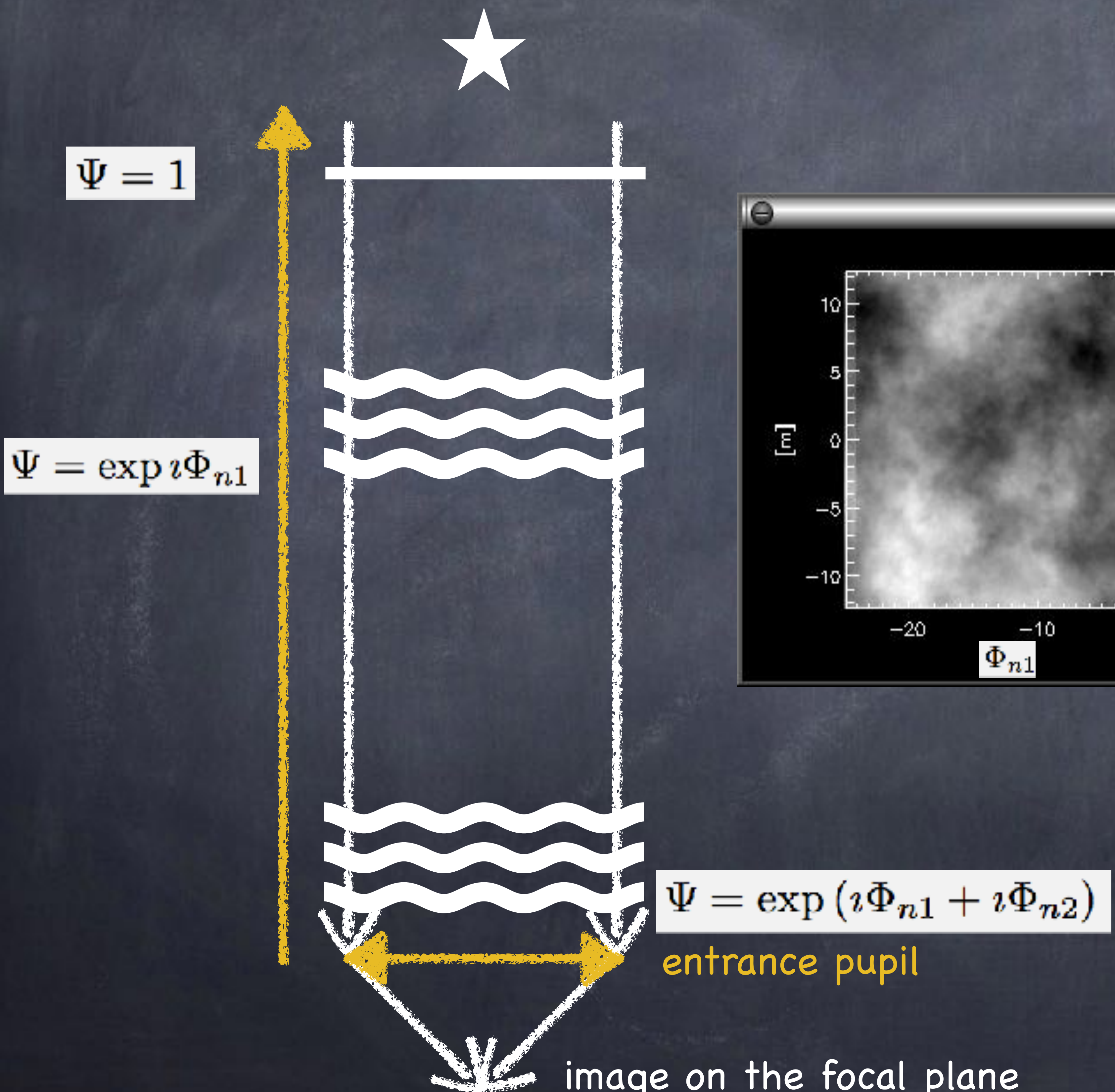

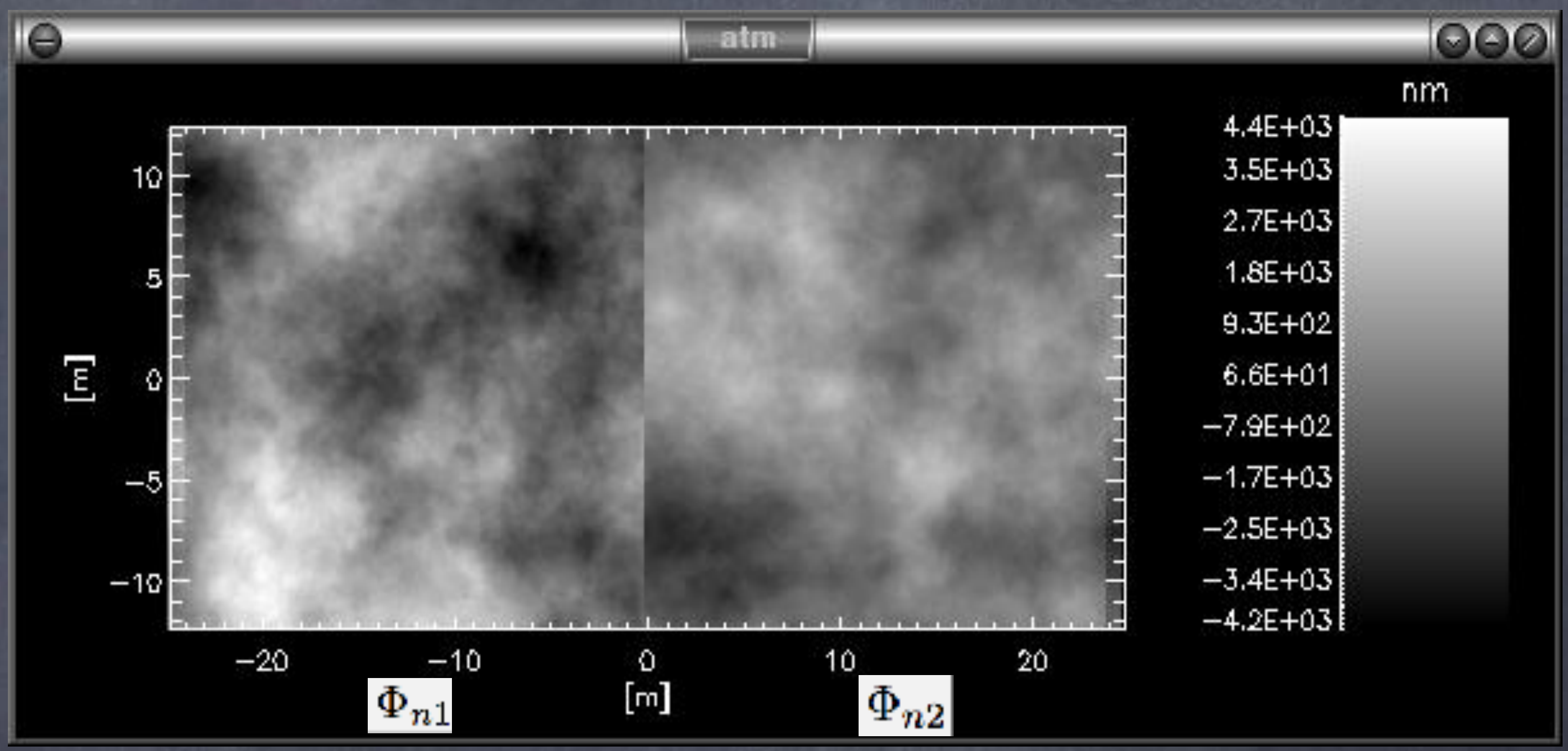

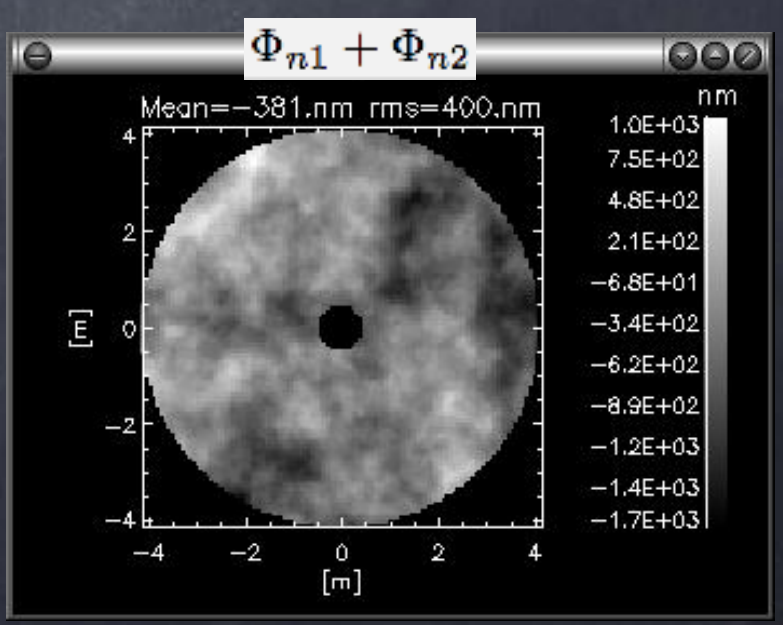

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entrance pupil

image on the focal plane

Images & turbulence - 8

The wavefront is, modulo $\lambda/2\pi$, proportional to the phase $\phi(r)$ of the wave $\psi(r)$ which has went through the turbulent atmosphere before reaching the telescope:

$$\Psi(\vec{r}) = A(\vec{r}) \exp\{\imath\Phi(\vec{r})\}$$

This phase can be decomposed following a base of polynomials, for example Zernike ones:

$$\Phi(\vec{r}) = \sum_i a_i \, Z_i(\vec{r})$$

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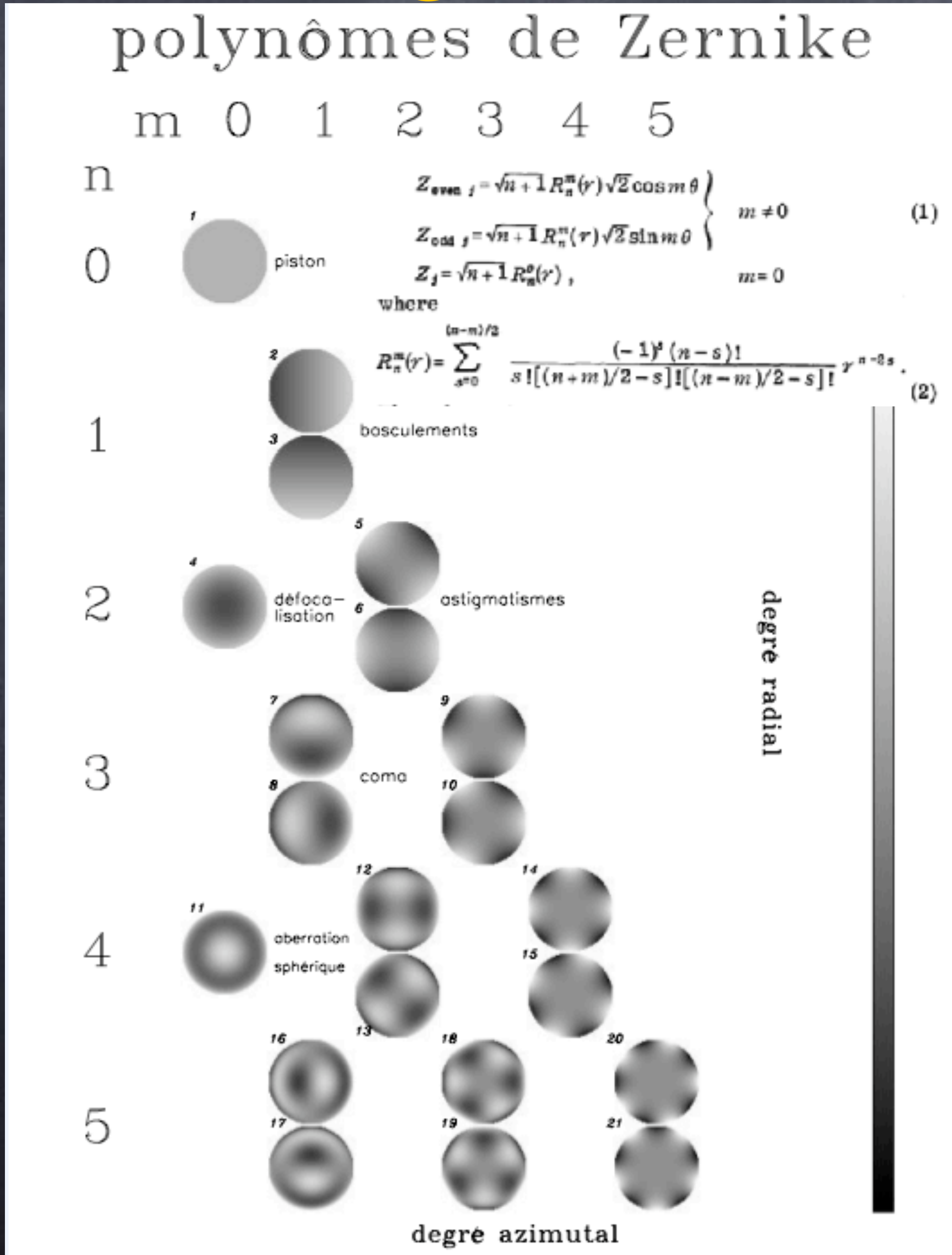
entrance pupil



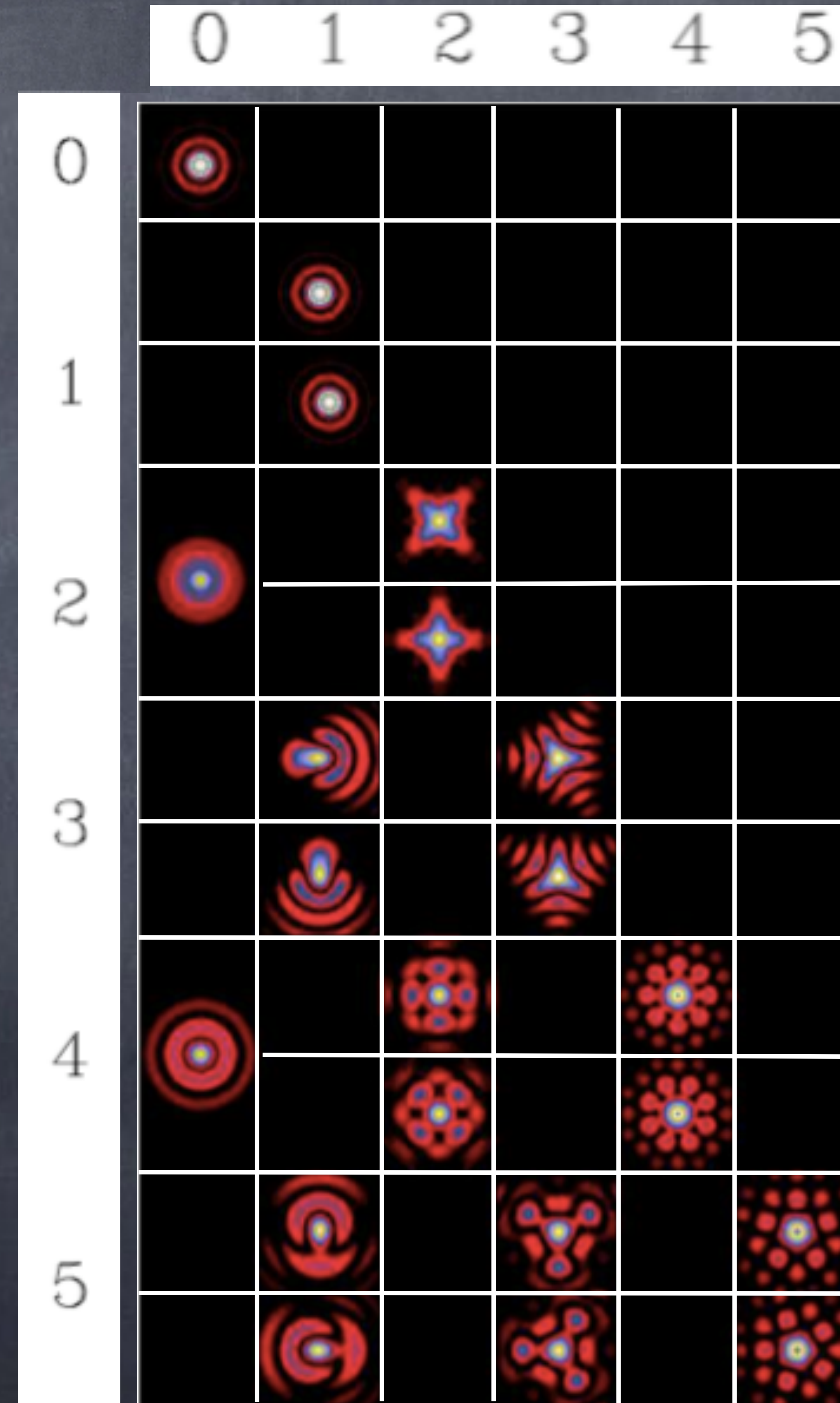


image on the focal plane

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turbulence intensity [$m^{1/3}$]

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

r_0 in band H knowing r_0 at 500nm (10cm) ?...

$$\tau = 0.36\ \frac{r_0}{v}$$

$$\epsilon = 0.98\ \frac{\lambda}{r_0}$$

$$\theta_0 = 0.314\ \frac{r_0}{\bar{h}}$$

$$\bar{v} = \left(\frac{\int C_n^2(h) v(h)^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

$$\bar{h} = \left(\frac{\int C_n^2(h) h^{\frac{5}{3}} dh}{\int C_n^2(h) dh}\right)^{\frac{3}{5}}$$

$$N_s \simeq 0.34 \left(\frac{D}{r_0}\right)^2$$

$$V_0 = c\ \tau_0\ r_0^2$$

volume of coherence

$$G_0 = r_0^2\ \tau_0\ \theta_0^2$$

coherence étendue

Number of speckles for r_0=10cm and D=1m ?...

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r_0 in band H knowing r_0 at 500nm ?...

$$r_0 = 0.185\ \lambda^{\frac{6}{5}}\ \cos(\gamma)^{\frac{3}{5}}\ \left[\int_0^\infty C_n^2(z)\ dz\right]^{-\frac{3}{5}}$$

$$r_0^{\mathrm{H}=1.65\,\mu\mathrm{m}} = r_0^{500\,\mathrm{nm}} \left(\frac{1.65}{0.5}\right)^{\frac{6}{5}} \simeq 0.42$$

Number of speckles for r_0=10cm and D=1m ?...

$$N_S^{500\,\mathrm{nm}} \simeq 0.34\ \left(\frac{1.0}{0.1}\right)^2 \simeq 34$$

$$N_S^{\mathrm{H}} \simeq 0.34\ \left(\frac{1.0}{0.42}\right)^2 \simeq 2$$

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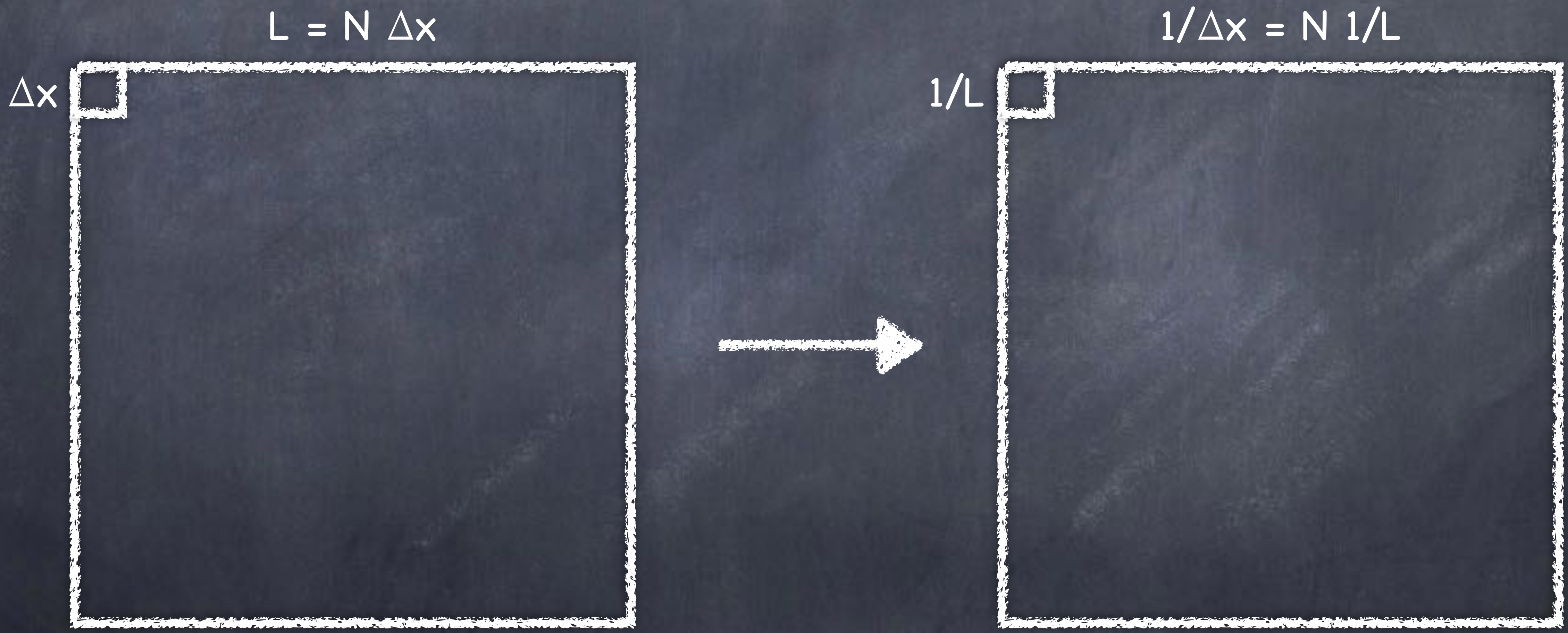
$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Power Spectral Density (PSD) of the phase, function of the spatial frequency

Kolmogorov/von Kármán model

- Kolmogorov : outerscale of turbulence $\mathcal{L}_0$ is infinite.
- One can refine the model by considering also ℓ_0.
- ∃ other models with a finite $\mathcal{L}_0$ and a non-zero ℓ_0.

(A reminder of discrete Fourier transform…)



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$$\Phi_\varphi(\vec{\nu}) = 0.0228\ r_0^{-\frac{5}{3}} \left(\nu^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-\frac{11}{6}}$$

Which, numerically written, and by considering wavefronts made of 'dim' pixels corresponding to 'L' meters, becomes:
(de-dimensionalizing the equation with L0=L0*L/L and f=f*L/L…)

```
freq = findgen(dim)
dsp  = .0228*(L/r0)^(5/3.)*L^2*(freq^2+(L/L0)^2)^(-11./6)
```

And which (with the right frequency scale) can be plot with:

```
plot_oo, 1./L*findgen(dim), dsp, XR=[1/L/1.2,dim*1/L*1.2], /XS
```

=> make a routine that computes PSD(L0,r0,dim,L) and plot it for different [r_0, L_0]… [with, for example: dim=1000, L=50, r0=0.1, L0=100,10,1]

-> Also read Aime (Sec. 1 & Sec. 2) and Maire (Chap.1)…