



# Initial velocity V-shapes of young asteroid families

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## ABSTRACT

Ejection velocity fields of asteroid families are largely unconstrained due to the fact that members disperse relatively quickly on Myr time-scales by secular resonances and the Yarkovsky effect. The spreading of fragments in  $a$  by the Yarkovsky effect is indistinguishable from the spreading caused by the initial ejection of fragments. By examining families <20 Myr old, we can use the V-shape identification technique to separate family shapes that are due to the initial ejection velocity field and those that are due to the Yarkovsky effect. Asteroid families that are <20 Myr old provide an opportunity to study the velocity field of family fragments before they become too dispersed. Only the Karin family's initial velocity field has been determined and scales inversely with diameter,  $D^{-1}$ . We have applied the V-shape identification technique to constrain young families' initial ejection velocity fields by measuring the curvature of their fragments' V-shape correlation in semimajor axis,  $a$ , versus  $D^{-1}$  space. Curvature from a straight line implies a deviation from a scaling of  $D^{-1}$ . We measure the V-shape curvature of 11 young asteroid families including the 1993 FY<sub>12</sub>, Aeolia, Brangane, Brasilia, Clarissa, Iannini, Karin, Konig, Koronis(2), Theobalda and Veritas asteroid families. We find that the majority of asteroid families have initial ejection velocity fields consistent with  $\sim D^{-1}$  supporting laboratory impact experiments and computer simulations of disrupting asteroid parent bodies.

**Key words:** minor planets – asteroids: general.

## 1 INTRODUCTION

Asteroid families are formed as a result of collisional disruptions and cratering events on larger parent bodies (e.g. Durda et al. 2004; Michel et al. 2015). Although dispersed in space, the family members typically cluster in their proper orbital elements, semimajor axis ( $a$ ), eccentricity ( $e$ ) and inclination ( $i$ ), close to that of the parent body (e.g. Hirayama 1918; Zappalà et al.; Nesvorný, Brož & Carruba 2015) and share similar spectral and reflectance properties (Cellino et al. 2002; Masiero et al. 2013; de León et al. 2016).

It was thought that asteroid fragments remained stationary in orbital elements space after their disruption (Zappalà et al. 1996; Cellino et al. 1999) requiring large ejection velocities to explain the wide dispersion of asteroid family fragments in orbital elements space. Impact simulations and asteroids observed on temporary, unstable orbits as well as family fragments leaking through mean motion and secular resonances provided evidence that asteroids' orbits were modified due to recoil from anisotropic surface emission of thermal photons, i.e. the Yarkovsky effect was responsible for the large dispersion of asteroids in orbital elements space (Michel

et al. 2001; Bottke et al. 2001). There is a degeneracy between the contribution of the initial spreading of an asteroid family fragments' orbital elements caused by the initial ejection of fragments and the contribution caused by the subsequent drift in  $a$  caused by the Yarkovsky effect. This degeneracy can only be broken in special cases, such as asteroids leaking through resonances or families that are too disperse to show the imprint of the initial ejection of fragments.

In addition to the cases above, young asteroid families can provide an opportunity to determine how ejection velocities of asteroid family fragments are distributed as a function of their size. The ejection velocities of Karin and Koronis asteroid family fragments have been measured to scale inversely with  $D$  by the use of Gauss' equations (Zappalà et al. 1990) or by linking the  $i$  distribution of family fragments to their out of plane velocities (Nesvorný et al. 2002; Carruba, Nesvorný & Aljbaae 2016a). In this paper, we study the least dispersed families in the asteroid belt, which most likely are the youngest, and examine how the spreading of their members depends on size. In an accompanying paper (Bolin et al. 2017), we show that semimajor axis spreading of the oldest and most disperse families follows a different dependence with the fragments' size. Thus, we demonstrate that the shape of a family in the  $a$  versus  $\frac{1}{D}$  space is a generic way to break the degeneracy between initial

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ejection velocity and Yarkovsky evolution and can be used to tell the relative contribution of each of these processes.

## 2 INITIAL VELOCITY FIELD V-SHAPES

The displacement in  $\Delta a = |a - a_c|$  after the disruption of a parent body, where  $a_c$  is the location in  $a$  of the parent body, is a function of  $V_T$ , the transverse velocity component of the ejected fragment and its parent body's mean motion,  $n$  (Zappalà et al. 1996)

$$|a - a_c| = \frac{2}{n} V_T. \quad (1)$$

The initial ejection of the family fragments should result in a symmetric V-shaped spread of fragments in  $a$  versus the reciprocal of the diameter,  $\frac{1}{D} = D_r$ , space because  $V_T$  scales inversely with asteroid diameter  $D$  (Cellino et al. 1999; Vokrouhlický et al. 2006a)

$$|a - a_c| = \frac{2}{n} V_{EV} \left( \frac{D_0}{D} \right)^{\alpha_{EV}} \cos(\theta) \quad (2)$$

$D_0$  is equal to 1329 km, where  $V_{EV}$  is a parameter that describes the width of the fragment ejection velocity distribution (Michel, Benz & Richardson 2004; Nesvorný et al. 2006; Vokrouhlický et al. 2006a,b; Durda et al. 2007).  $V_{EV}$  for known asteroid families such as Karin and Erigone range between 15 and 50 m s<sup>-1</sup> (Nesvorný et al. 2006; Bottke, Vokrouhlický & Nesvorný 2007; Carruba & Morbidelli 2011; Masiero et al. 2012; Nesvorný, Brož & Carruba 2015).

$V_{EV}$  is determined by modelling the initial ejection of fragments according to equation (2), where  $\alpha_{EV}$  is the exponent scaling  $V_{EV}$  with  $D$  (Vokrouhlický et al. 2006a,b).  $\alpha_{EV} = 1$  would imply a simple  $\frac{1}{D}$   $V_{EV}$  dependence. Modelling of the ejection of fragments is done for asteroid families younger than  $\lesssim 20$  Myr where the Yarkovsky effect has not had enough time to modify the  $a$  of the fragments such as for the Karin asteroid family (Nesvorný et al. 2006). The modelling of the initial ejection velocities of the fragments includes evolution of family fragments'  $a$  according to the Yarkovsky and YORP effects (Bottke et al. 2006; Vokrouhlický et al. 2015). Additional constraints to the initial ejection velocity field can be provided by the  $i$  distribution of an asteroid family, which is supposed to remain essentially unaltered during the Yarkovsky evolution (Carruba & Nesvorný 2016; Carruba et al. 2016a). For cases where  $V_{EV}$  cannot be determined, the escape velocity of the asteroid family parent body is used for  $V_{EV}$  (Vokrouhlický et al. 2006a; Walsh et al. 2013). The escape velocity of the parent body is a good estimate for  $V_{EV}$  because most particles are ejected from their parent bodies at velocities around the escape velocity of the parent body in numerical impact simulations (Durda et al. 2007; Ševeček et al. 2017).

Karin and Koronis asteroid family data suggest that  $\alpha_{EV} \simeq 1.0$  (Nesvorný et al. 2002, 2006; Carruba et al. 2016a) while analytical calculations of  $\alpha_{EV}$  can be as high as  $\simeq 1.5$  (Cellino et al. 1999).  $\cos(\theta)$  is the angle of the fragment's velocity relative the transverse direction of the parent body's orbit. In equation (2),  $\cos(\theta)$  is expected to be uniformly distributed between  $-1$  and  $1$ . We assume that the number of fragments is high enough that the V-shape's edge is defined by fragments with  $\cos(\theta) \simeq 1.0$  or  $-1.0$  in equation (2). The  $V_{EV}$  of fragments interior to the V-shape border with the same value of  $|\cos(\theta)| < 1$  will scale with  $D$  by the same  $\alpha_{EV}$  as fragments on the V-shape border. The left-hand side of equation (2) is decreased for V shapes with fewer fragments by a factor of  $\frac{2}{\pi}$  because  $\frac{2}{\pi}$  is the average value of  $\cos(\theta)$  between the intervals  $0$  and  $\frac{\pi}{2}$  and  $\frac{\pi}{2}$  and  $\pi$  resulting in a distorted value of  $\alpha$ .

We re-write equation (2) in  $a$  versus  $D_r$  space with  $\cos(\theta) = 1.0$  to obtain  $D_r$  as a function of  $a, a_c, n, V_{EV}$  and  $\alpha_{EV}$

$$D_r(a, a_c, n, V_{EV}, \alpha_{EV}) = \frac{1}{D_0} \left( \frac{|a - a_c| n}{2 V_{EV}} \right)^{\frac{1}{\alpha_{EV}}}. \quad (3)$$

The spread in family fragments by their initial ejection from the parent body has the same functional form of the spreading of family fragments caused by the Yarkovsky effect (Bottke et al. 2006; Vokrouhlický et al. 2015).

$$D_r(a, a_c, C_\alpha, p_V, \alpha) = \frac{\sqrt{p_V}}{D_0} \left( \frac{|a - a_c|}{C_\alpha} \right)^{\frac{1}{\alpha}}. \quad (4)$$

$C_\alpha$  is the width of the V-shape for a specific value of  $\alpha$ .  $C_\alpha$  is normalized to the width of the V-shape with  $\alpha = 1.0$

$$C = C_\alpha \left( \frac{\sqrt{p_V}}{1329} \right)^{1-\alpha}. \quad (5)$$

Equation (4) is re-written in terms of  $(a, a_c, C, p_V, \alpha)$  by using equation (5)

$$D_r(a, a_c, C, p_V, \alpha) = \left( \frac{|a - a_c| \sqrt{p_V}}{D_0 C} \right)^{\frac{1}{\alpha}} \quad (6)$$

The parameters,  $\alpha_{EV}$  and  $\alpha$  in equations (2) and (6), describing the shape of family V shapes in  $a$  versus  $D_r$ , caused by the  $D$  dependent initial ejection of fragments or Yarkovsky force are functionally equivalent.

We combine equations (3) and (6) to obtain the spread of the V shape,  $C$ , caused by the initial ejection of fragments as a function of  $n, V_{EV}, p_V$  and  $\alpha_{EV}$

$$C(n, V_{EV}, p_V, \alpha_{EV}) = \frac{2 \cdot D_0^{\alpha_{EV}-1}}{n} V_{EV} \sqrt{p_V}. \quad (7)$$

We apply the techniques to identify asteroid family Yarkovsky V-shapes as defined in Bolin et al. (2017) to measure the  $\alpha$  of an asteroid family's initial ejection velocity V-shape because the shape of the initial ejection velocity field as described by equation (3) is similar to shape acquired by the Yarkovsky effect described by equation (6).

### 2.1 V-shape identification technique and measurement of $\alpha$

The identification of the family V-shape is performed by determining  $a_c, C$  and  $\alpha$  for a family V-shape according to equation (6) in  $a$  versus  $D_r$  space using a modified version of the border method from Bolin et al. (2017).

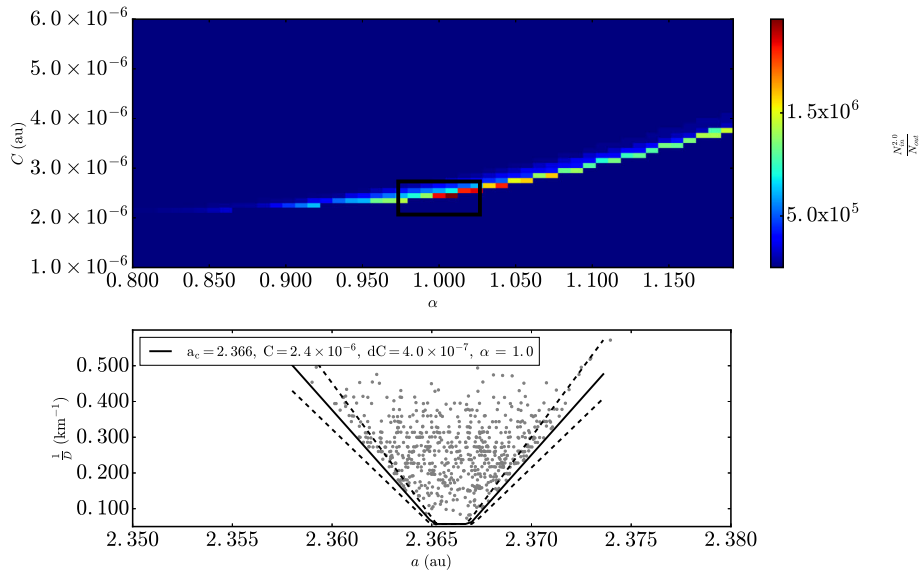
$N_{out}(a_c, C, dC, p_V, \alpha)$

$$\begin{aligned} & \frac{\sum_j w(D_j) \int_{a_1}^{a_2} da \int_{D_r(a, a_c, C, p_V, \alpha)}^{D_r(a, a_c, C, p_V, \alpha)} dD_r \delta(a_j - a) \delta(D_{r,j} - D_r)}{\int_{a_1}^{a_2} da \int_{D_r(a, a_c, C, p_V, \alpha)}^{D_r(a, a_c, C, p_V, \alpha)} dD_r}, \quad (8) \end{aligned}$$

$N_{in}(a_c, C, dC, p_V, \alpha)$

$$\begin{aligned} & \frac{\sum_j w(D_j) \int_{a_1}^{a_2} da \int_{D_r(a, a_c, C, p_V, \alpha)}^{D_r(a, a_c, C, p_V, \alpha)} dD_r \delta(a_j - a) \delta(D_{r,j} - D_r)}{\int_{a_1}^{a_2} da \int_{D_r(a, a_c, C, p_V, \alpha)}^{D_r(a, a_c, C, p_V, \alpha)} dD_r}. \quad (9) \end{aligned}$$

Equations (8) and (9) are normalized the area in  $a$  versus  $D_r$  between the nominal and outer V-shapes defined by  $D_r(a, a_c, C_+, p_V, \alpha)$  and



**Figure 1.** Application of the V-shape identification to synthetic asteroid family data at time = 0. Top panel: the ratio of  $N_{\text{out}}(a_c, C, dC, p_V, \alpha)^2$  to  $N_{\text{in}}(a_c, C, dC, p_V, \alpha)$  ratio in the  $\alpha$ - $C$  range,  $(a_c \pm \frac{\Delta\alpha}{2}, C \pm \frac{\Delta C}{2})$ , where  $\Delta\alpha$  is equal to  $8.0 \times 10^{-3}$  au and  $\Delta C$ , not to be confused with  $dC$ , is equal to  $1.0 \times 10^{-9}$  au, for the single synthetic family. The box marks the peak value in  $\frac{N_{\text{in}}(a_c, C, dC, p_V, \alpha)^2}{N_{\text{out}}(a_c, C, dC, p_V, \alpha)}$  for the synthetic family V-shape. Bottom panel:  $D_r(a, a_c, C, p_V, \alpha)$  is plotted for the peak value with the primary V-shape as a solid line where  $p_V = 0.05$ . The dashed lines mark the boundaries for the area in  $a$  versus  $D_r$  space for  $N_{\text{in}}$  and  $N_{\text{out}}$  using equation (6),  $D_r(a, a_c, C \pm dC, p_V, \alpha)$ , where  $a_c = 2.366$  au and  $dC = 4.0 \times 10^{-7}$  au.

$D_r(a, a_c, C, p_V, \alpha)$  in the denominator for equation (8) and between the nominal and inner V-shapes defined by defined by  $D_r(a, a_c, C, p_V, \alpha)$  and  $D_r(a, a_c, C_-, p_V, \alpha)$  in the denominator for equation (9).

The symbol  $\Sigma_j$  in equations (8) and (9) indicates summation on the asteroids of the catalogue, with semimajor axis  $a_j$  and reciprocal diameter  $D_{r,j}$ . The symbol  $\delta$  indicates Dirac's function, and  $a_1$  and  $a_2$  are the low- and high-semimajor axes range in which the asteroid catalogue is considered. The function  $w(D)$  weighs the right-hand side portions of equations (8) and (9) by their size so that the location of the V-shape in  $a$  versus  $D_r$  space will be weighted towards its larger members. The exponent 2.5 is used for  $w(D) = D^{2.5}$ , in agreement with the cumulative size distribution of collisionally relaxed populations and with the observed distribution for MBAs in the  $H$  range  $12 < H < 16$  (Jedicke, Larsen & Spahr 2002).

Walsh et al. (2013) found that the borders of the V-shapes of the Eulalia and new Polana family could be identified by the peak in the ratio  $\frac{N_{\text{in}}}{N_{\text{out}}}$ , where  $N_{\text{in}}$  and  $N_{\text{out}}$  are the number of asteroids falling between the curves defined by equation (6) assuming  $\alpha = 1.0$  for values  $C$  and  $C_-$  and  $C$  and  $C_+$ , respectively, with  $C_- = C - dC$  and  $C_+ = C + dC$ . We extend our technique to search for a peak in the ratio  $\frac{N_{\text{in}}^2}{N_{\text{out}}}$ , which corresponds to weighting the ratio of  $\frac{N_{\text{in}}}{N_{\text{out}}}$  by the value of  $N_{\text{in}}$ . This approach has been shown to provide sharper results (Delbo et al. 2017). Here, we extend the search for a maximum of  $\frac{N_{\text{in}}^2}{N_{\text{out}}}$  to three dimensions, in the  $a_c$ ,  $C$  and  $\alpha$  space. A peak value in  $\frac{N_{\text{in}}(a_c, C, dC, p_V, \alpha)^2}{N_{\text{out}}(a_c, C, dC, p_V, \alpha)}$  as seen in the top panel of Fig. 1 for the synthetic asteroid family described in Section 3.1. For simplicity only the projection on the  $\alpha$ ,  $C$  plane compared to Bolin et al. (2017) and Delbo et al. (2017) that used only the projection in the  $a_c$ ,  $C$  plane) indicates the best-fitting values of  $a_c$ ,  $C$  and  $\alpha$  for a family V-shape using equation (6) (bottom panel of Fig. 1).

The value of  $dC$  is used similarly as in Bolin et al. (2017). The value of  $dC$  used depends on the density of asteroids on the family V-shape edge. The value of  $dC$  can be a few 10 percent of the V-shape's  $C$  value if the density of asteroids on the V-shape edge

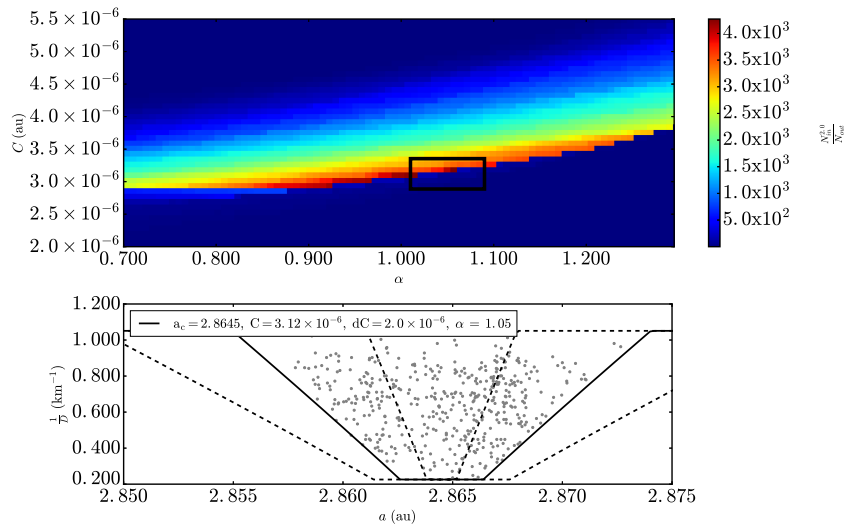
is high such as the case of the Karin family (see the bottom panel of Fig. 2) and more, up to 40 ~ 50 percent if the V-shape edge is more diffuse such as in the case of the Brangane family (see the bottom panel of Fig. 3) (Milani et al. 2014; Nesvorný, Brož & Carruba 2015; Spoto, Milani & Knežević 2015). The inner and outer V-shapes must be wide enough to include enough asteroids in the inner V-shape and measure a  $N_{\text{in}}^2$  to  $N_{\text{out}}$  ratio high enough to identify the family V-shape. The V-shape can include interlopers or asteroids that are not part of the family V-shape if the value used for  $dC$  is used is too large (Nesvorný et al. 2015; Radović et al. 2017). The V-shape identification technique was tested on families identified by both Nesvorný et al. (2015) and Milani et al. (2014) to verify that the V-shape  $a_c$ ,  $C$  and  $\alpha$  determination works on family membership definitions from either data base and produces similar results. The V-shape  $a_c$ ,  $C$  and  $\alpha$  determination technique was tested the 1993 FY<sub>12</sub>, Aeolia, Brangane, Brasilia, Iannini and König families identified by both Nesvorný et al. (2015) and Milani et al. (2014) discussed in Sections A1, A2, A3, A4, A6 and A8.

## 2.2 Contribution to width of young asteroid family V-shapes by the Yarkovsky effect

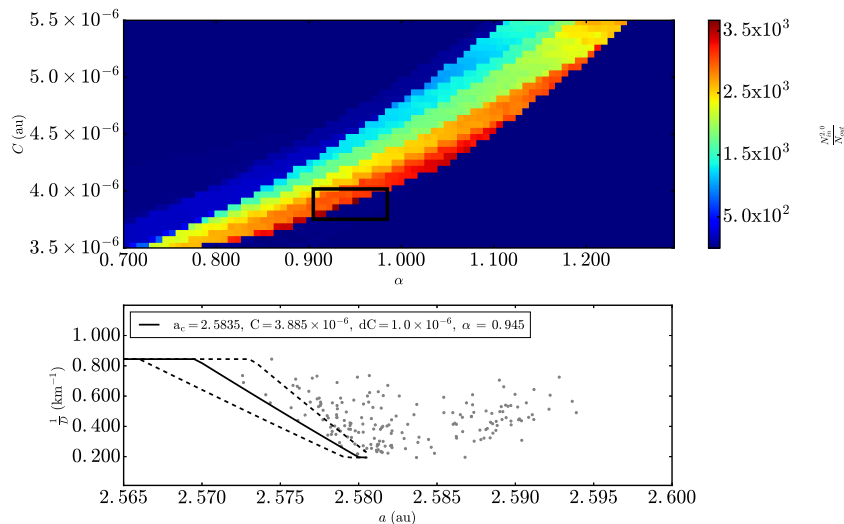
The value  $C$  obtained by the V-shape  $a_c$ ,  $C$  and  $\alpha$  determination method in Section 2.1 includes the contribution of the initial ejection Velocity field from equation (7) and the contribution to  $C$  from the Yarkovsky effect (Vokrouhlický et al. 2006a; Nesvorný et al. 2015)

$$C = C_{\text{YE}} + C_{\text{EV}}, \quad (10)$$

where  $C_{\text{YE}}$  is the width of the V-shape due to the Yarkovsky effect and  $C_{\text{EV}}$  is the width of the V-shape due to the initial ejection velocity of fragments. Asteroid family V-shapes with  $C = C_{\text{EV}}$  only in equation (10) are indistinguishable from asteroid families, which have contribution to their value of  $C$  from both the Yarkovsky effect and ejection velocity. The nominal value of  $C_{\text{EV}}$  exceed more



**Figure 2.** The same as Fig. 1 for Karin asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3}$  au and  $\Delta C$ , is equal to  $8.0 \times 10^{-9}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.21$ ,  $a_c = 2.865$  au and  $dC = 2.0 \times 10^{-6}$  au.



**Figure 3.** The same as Fig. 1 for Brangane asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3}$  au and  $\Delta C$ , is equal to  $5.5 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.1$ ,  $a_c = 2.584$  au and  $dC = 1.0 \times 10^{-6}$  au.

than 50 per cent of  $C$  for asteroid families younger than 100 Myr (Nesvorný et al. 2015; Carruba & Nesvorný 2016). The error on the parent body size and the resultant calculation of  $C$  from the parent body’s escape velocity can be large enough so that there is a possibility that  $C - C_{EV} \lesssim 0$ . In this work, we select for analysis all the families for which  $C - C_{Ejection\ velocity} \lesssim 0$ , considering that these families are young enough that the contribution of the Yarkovsky effect to the spread in  $a$  of the family fragments is minimal and therefore we assume that the value of  $C$  obtained with the techniques in Section 2.1 is equal to  $C_{EV}$ .

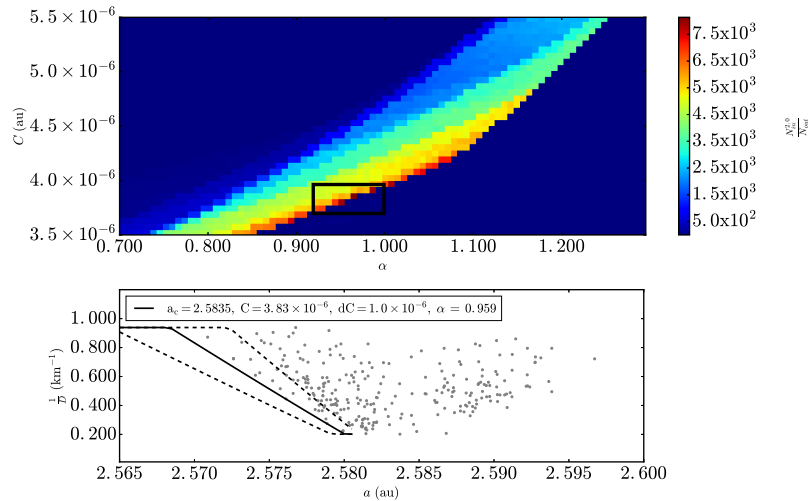
However, it should be noted that the Yarkovsky effect still affects the displacement in  $a$  of even young family members. Nesvorný & Bottke (2004) and Carruba, Nesvorný & Vokrouhlický (2016b) showed that the Yarkovsky drift rate and displacement in  $a$  could be determined for Karin family fragments by backwards integrating the orbits of the family fragments backwards in time and measuring the convergence of the ascending node,  $\Omega$ , and longitude of perihelion,  $\omega$ , of the Karin family fragments relative to asteroid Karin while

under the influence of the Yarkovsky effect. Although the effect of the Yarkovsky force on the Karin family fragments is strong enough to be detected using the techniques in Nesvorný & Bottke (2004) and Carruba et al. (2016b), the displacement in  $a$  over the age of the young Karin family is not large enough to affect our assumptions (see Section A7 about the Karin family as an example).

### 2.3 Data set and uncertainties of $\alpha$ measurements

#### 2.3.1 Data set

The data used to measure the V-shapes of asteroid family were taken from the MPC catalogue for the  $H$  magnitudes. Family definitions were taken from Nesvorný et al. (2015). Asteroid family data for the 1993 FY<sub>12</sub>, Brangane, and Iannini families were used from both Milani et al. (2014) and Nesvorný et al. (2015) to verify that the V-shape technique provided similar results for  $a_c$ ,  $C$  and  $\alpha$  for the same family with asteroid data taken from different



**Figure 4.** Same as Fig. 3, but repeated for the Brangane family defined by Milani et al. (2014)

asteroid family data bases. Results from the V-shape technique using asteroid family data from Nesvorný et al. (2015) were repeated with asteroid family data from Milani et al. (2014) for the 1993 FY<sub>12</sub>, Aeolia, Brangane, Brasilia, Iannini and König asteroid families as seen for the Brangane family in Figs 3 and 4. Family visual albedo,  $p_V$ , data from Masiero et al. (2013) and Spoto et al. (2015) were used to calibrate the conversion from  $H$  magnitudes into asteroid  $D$  using the relation  $D = 2.99 \times 10^8 \frac{10^{0.2(m_\odot - H)}}{\sqrt{p_V}}$  (Bowell et al. 1988), where  $m_\odot = -26.76$  (Pravec & Harris 2007). Numerically and analytically calculated MBA proper elements were taken from the Asteroid Dynamic Site<sup>1</sup> (Knežević & Milani 2003). Numerically calculated proper elements were used preferentially and analytical proper elements were used for asteroids, that had numerically calculated elements as of April 2017.

### 2.3.2 Uncertainty of $\alpha$

The value of  $\alpha$  located where  $\frac{N_{in}(a_c, C, dC, p_V, \alpha)^2}{N_{out}(a_c, C, dC, p_V, \alpha)}$  peaks in  $\alpha$  versus  $C$  space represents the best estimate of the  $\alpha$  of an asteroid family's V-shape using the nominal  $a$  and  $D_r$  asteroid values. Different values in the physical properties of asteroids cause a spread in possible  $\alpha$  values that measured together are the uncertainty in the measured value of  $\alpha$ . Changes in asteroids'  $D$  is caused by variations in their  $H$  magnitude measurements and spread in  $p_V$  during the conversion of asteroid  $H$  into  $D$ . In addition to the variety of different possible  $D$  values for asteroids and a lack of complete information about the true population of asteroids within a family, the contribution of outliers to a family's  $a$  versus  $D_r$  distribution can increase the spread in  $\alpha$  values compatible with the family V-shape. We devise the following Monte Carlo procedure to quantify the spread in  $\alpha$  measurements of family V-shapes caused by random differences in the asteroid physical properties between family members and incomplete information about the asteroid family member population.

At least 1200 Monte Carlo trials are completed per family. Some families have significantly more than 1200 Monte Carlo trials as described in the Appendix if additional CPU time was available. In each trial, the location of the peak  $\frac{N_{in}(a_c, C, dC, p_V, \alpha)^2}{N_{out}(a_c, C, dC, p_V, \alpha)}$  value in  $\alpha$  versus  $C$  is recorded. Three steps are completed to randomize the asteroid family data from the original  $a$  versus  $D_r$  distribution per

trial. The first step is to create a resampled data set of family fragments by removing  $\sqrt{N}$  objects randomly, where  $N$  is the number of objects in  $a$  versus  $D_r$  space to include variations caused by incomplete knowledge of the asteroid family fragment population. Incompleteness of asteroid family fragments increases for smaller fragments and is more pronounced in the middle and outer portions of the Main Belt (Jedicke & Metcalfe; Jedicke et al. 2002). The variation of  $\alpha$  caused by the incomplete knowledge of the family fragment population is more weighted towards smaller fragments than larger fragments as a result of the increased incompleteness and greater number of smaller Main Belt asteroids in the asteroid family catalogues

This second step is taken to determine the variation caused by incomplete information in the family fragment population by re-sampling the fragments'  $a$  by their own  $a$  distribution per  $D_r$  bin. In this step, family fragments are randomized by the semimajor axis distribution of fragments in each  $D_r$  bin with a size of  $0.001 \text{ km}^{-1}$ .

The third step is to randomize the measurements of  $H$  and  $p_V$  of the asteroids by their known uncertainties. Asteroid  $H$  values were randomized between 0.2 and 0.3 magnitudes known uncertainties for  $H$  values from the Minor Planet Center Catalogue (Oszkiewicz et al. 2011; Pravec et al. 2012) with an average offset of 0.1 magnitudes consistent for asteroids with  $12 < H < 18$  (Pravec et al. 2012; Vereš et al. 2015). After the  $H$  values are randomized, asteroid fragments'  $H$  were converted into  $D$  using the relation

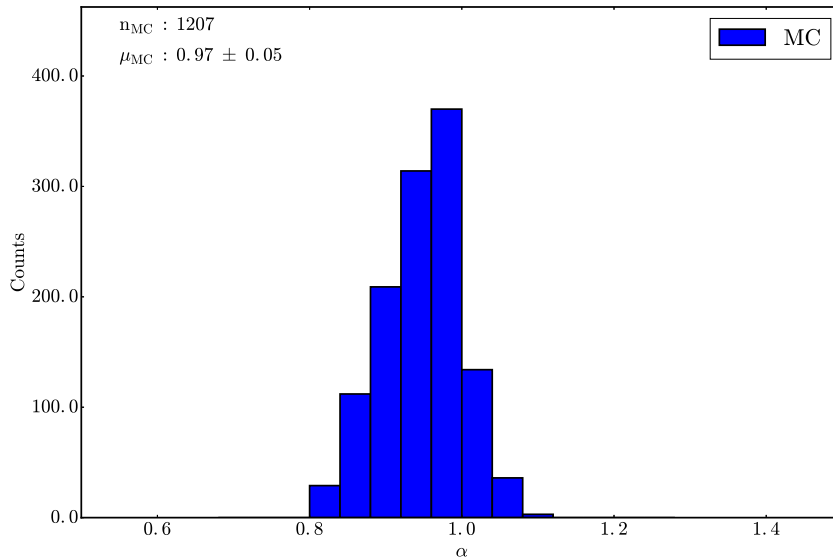
$$D = 2.99 \times 10^8 \frac{10^{0.2(m_\odot - H)}}{\sqrt{p_V}} \quad (11)$$

from Harris & Lagerros (2002), and a value of  $p_V$  chosen at random for each asteroid using central values and uncertainties per asteroid family from Masiero et al. (2013) and Spoto et al. (2015).

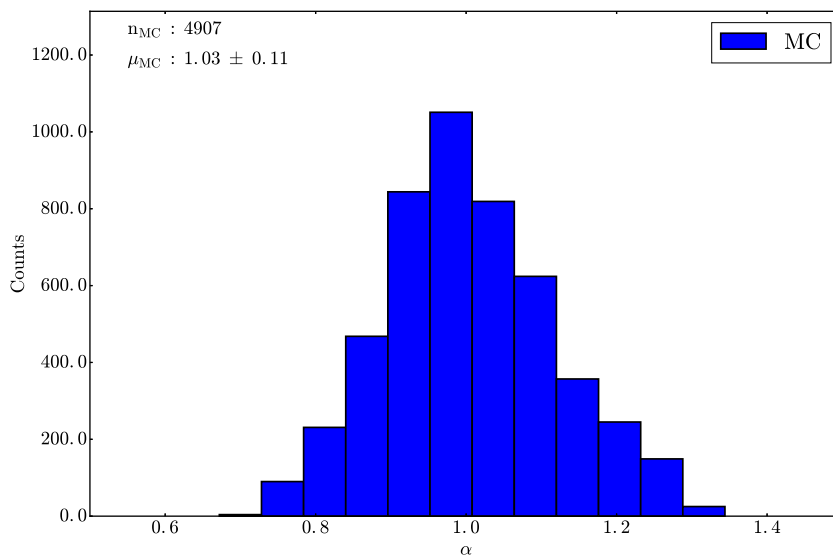
The mean and root-mean-square (RMS) uncertainty of  $\alpha$  was determined from the distribution of  $\alpha$  in the Monte Carlo trials. Having more fragments and a well-defined V-shape causes the Monte Carlo technique to produce a narrower distribution in  $\alpha$  (e.g. for the Karin family,  $\alpha = 0.97 \pm 0.05$ , Fig. 5), while having fewer fragments and a more diffuse V-shape results in a broader  $\alpha$  distribution (e.g. for the 1993 FY<sub>12</sub> family,  $\alpha = 1.03 \pm 0.11$ , Fig. 6).

<sup>1</sup> <http://Hamilton.dm.unipi.it/astdys/>





**Figure 5.** Histogram of  $\alpha$  located at the peak value of  $N_{\text{out}}(a_c, C, dC, p_V, \alpha)^2$  to  $N_{\text{in}}(a_c, C, dC, p_V, \alpha)$  in each of the  $\sim 1200$  trials repeating the V-shape technique for the Karin family. The mean of the distribution is centred at  $\alpha = 0.97 \pm 0.05$  and the bin size in the histogram is 0.04 consistent with Nesvorný et al. (2002).



**Figure 6.** The same as Fig. 5 with  $\sim 4900$  trials repeating the V-shape technique for the 1993 FY<sub>12</sub> family. The mean of the distribution is centred at  $\alpha = 1.03 \pm 0.11$  and the bin size in the histogram is 0.06.

### 3 RESULTS

#### 3.1 Synthetic family

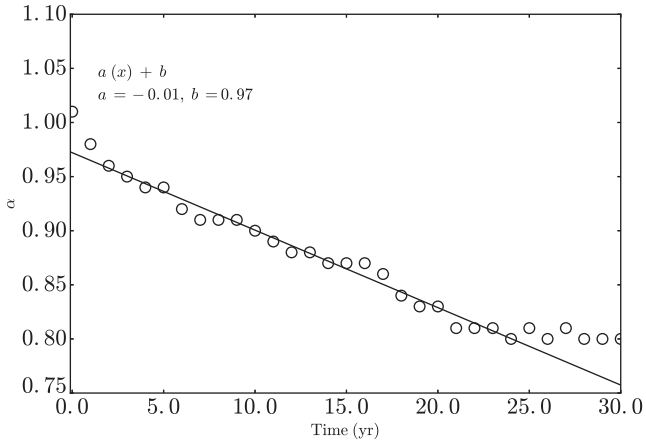
It is generally expected that  $\alpha_{\text{EV}} \simeq 1$  as discussed in Section 2. Recent work on the V-shapes of asteroid families  $>100$  Myr old suggests that  $\alpha_{\text{YE}} \simeq 0.8$  (Bolin et al. 2017). The time it takes to transition from a V-shape having its  $\alpha$  equal to  $\alpha_{\text{EV}} = 1.0$  to  $\alpha$  equal to  $\alpha_{\text{YE}} \simeq 0.8$  is the time it takes for families to have their initial ejection velocity fields erased by the Yarkovsky effect. We determine the time when the Yarkovsky effect erases  $\alpha_{\text{EV}}$  by simulating the initial ejection of fragments from the disruption of a parent body and their subsequent spreading caused by the Yarkovsky effect.

The break up of a synthetic asteroid family and its fragments' subsequent evolution due to the Yarkovsky effect is simulated by using 650 particles at  $(a, e, \sin i) = (2.37, 0.21, 0.08)$  and distributed

in  $a$  versus  $D_r$  space according to equation (2) with  $\alpha_{\text{EV}} = 1.0$  and  $V_{\text{EV}} = 30 \text{ m s}^{-1}$  using fragments with  $2 < D < 75 \text{ km}$  distributed according to the known members of the Erigone family defined by Nesvorný et al. (2015). The eccentricity and inclination distributions were determined by using Gaussian scaling described in Zappalà et al. (2002).  $V_{\text{EV}} = 30 \text{ m s}^{-1}$  corresponds to a typical initial displacement of  $\sim 7.0 \times 10^{-3} \text{ au}$  for a 5 km diameter asteroid. The Yarkovsky drift rates were defined with

$$\frac{da}{dt}(D, \alpha, a, e, N, A) = \left(\frac{da}{dt}\right)_0 \frac{\sqrt{a_0(1-e_0^2)}}{\sqrt{a(1-e^2)}} \left(\frac{D_0}{D}\right)^{\alpha_{\text{YE}}} \times \left(\frac{\rho_0}{\rho}\right) \left(\frac{1-A}{1-A_0}\right) \left(\frac{\text{au}}{\text{Myr}}\right) \frac{\cos(\theta)}{\cos(\theta_0)} \quad (12)$$

from Spoto et al. (2015) and Bolin et al. (2017) with  $\left(\frac{da}{dt}\right)_0 \sim 4.7 \times 10^{-5} \text{ au Myr}^{-1}$ ,  $a_0 = 2.37 \text{ au}$ ,  $e_0 = 0.2$ ,  $D_0 = 5 \text{ km}$ ,



**Figure 7.** Time versus  $\alpha$  for the first 30 Myr of time evolution of the fragments of a synthetic asteroid family given an  $\alpha_{\text{EV}} = 1.0$  according to equation (2) and evolved in time  $\alpha_{\text{YE}} = 0.8$  according to equation (12). The steep decrease in  $\alpha$  after  $\sim 0$  Myr is due to the randomization of asteroid fragment obliquity and subsequent evolution of their  $a$ 's according to equation (12). The dark line is a linear fit to the first 20 Myr of simulation data.

$\rho_0 = 2.5 \text{ g cm}^{-3}$  and bond albedo,  $A_0$ , is equal to 0.1, surface conductivity between 0.001 and 0.01  $\text{W m}^{-1} \text{K}^{-1}$  and  $\theta_0 = 0^\circ$  (Bottke et al. 2006; Vokrouhlický et al. 2015). For the synthetic family,  $\rho = 2.3 \text{ g cm}^{-3}$ ,  $A = 0.02$  and  $\cos(\theta)$  is uniformly distributed between  $-1$  and  $1$ . An  $\alpha_{\text{YE}} = 0.8$  was chosen as  $\alpha$  measurements of asteroid families old enough to have their fragments significantly modified by the Yarkovsky effect have  $0.7 \lesssim \alpha \lesssim 0.9$  (Bolin et al. 2017). The particles were evolved with the Yarkovsky effect and gravitational perturbations from Mercury, Venus, Earth, Mars, Jupiter and Saturn using the SWIFT\_RMVS code (Levison & Duncan 1994). Particles are removed from the simulation if they collide with one of the planets or evolve on to small perihelion orbits. YORP rotational and spin-axis variation are not included in the simulation.

The V-shape identification technique was applied on the synthetic family data at time = 0 by using the techniques in Section 2.1. Equations (8) and (9) are integrated using the interval  $(-\infty, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  and the interval  $[0.04, 0.60]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$ . Equation (6) is truncated to 0.04 for  $D_r < 0.04$  and to 0.60 for  $D_r > 0.60$ . Asteroids with  $0.04 < D_r < 0.60$  were chosen because the number of asteroids in this  $D_r$  is large enough so that the leading edge of the V-shape is defined by asteroids with  $\cos(\theta) = 1.0$  or  $-1.0$  according to equation (2).

The V-shape identification technique located a peak at  $(a_c, C, \alpha) = (2.366 \text{ au}, 2.4 \times 10^{-6} \text{ au}, \sim 1.0)$  as seen in the top panel of Fig. 1. The peak value of  $\frac{N_{\text{in}}^2}{N_{\text{out}}}$  is  $\sim 11$  standard deviations above the mean value of  $\frac{N_{\text{in}}^2}{N_{\text{out}}}$  in the range  $2.35 < a < 2.38 \text{ au}$ ,  $1.0 \times 10^{-6} < C < 6.0 \times 10^{-6} \text{ au}$  and  $0.8 < \alpha < 1.2$ . A  $dC = 4.0 \times 10^{-7} \text{ au}$  was used. The concentration of the peak to one localized area in  $\alpha$  versus  $C$  space is due to the sharpness of the synthetic family's V-shape border.

The V-shape identification technique was applied to family fragments at 1 Myr steps for the first 100 Myr of the simulation. The value of  $\alpha$  for the V-shape linearly decreases below 1.0 and reaches  $\sim 0.8$ , equal to  $\alpha_{\text{YE}}$  after  $\sim 20$  Myr (see Fig. 7). There is a steep drop in  $\alpha$  from  $\sim 1.0$  to  $\sim 0.97$  in the first 1–2 Myr of the simulation that is possibly due to the fragments near the borders of the family becoming spread in  $a$  according to equation (12). This is because fragments at the border of the family with  $\cos(\theta) = \pm 1$  are

the Yarkovsky front-runners and cause a very quick spreading of the family, rapidly changing the value of  $\alpha$ . Assuming  $\alpha_{\text{YE}}$  equal to  $\sim 0.8$  for the Yarkovsky drift size dependence in the Main Belt, asteroid family V-shapes with a measured  $\alpha$  closer to 0.8 reveal that the dispersion of the family is dominated by the Yarkovsky effect over the initial ejection of fragments and older than  $\sim 20$  Myr. Family V-shapes that have measured  $\alpha$  values of  $\sim 1.0$  have the contribution of the initial ejection velocity field dominant to the value of  $\alpha$  and the measured value of  $\alpha$  is characteristic of the size-dependence of the initial ejection velocity.

### 3.2 Young asteroid family ejection velocity V-shapes

The V-shape identification technique was applied to 11 asteroid families noted for their young ages between a few Myrs to a few 10 Myr (Spoto et al. 2015; Nesvorný et al. 2015) listed in the first column of Table 2, selected for having  $C - C_{\text{EV}} \lesssim 0$  as explained in Section 2.2. The measured value of their V-shape's  $\alpha$  and uncertainties determined by the techniques in Sections 2.1 and 2.3.2 as well as physical properties used to measure the  $\alpha$  of family V-shapes are summarized in Table 2. A description of how the V-shape identification technique is implemented for each family is described in the Appendix.

## 4 DISCUSSION AND CONCLUSION

We have demonstrated that the techniques of Bolin et al. (2017) can be used not only to identify asteroid family V-shapes, but also to measure the spreading of family fragments caused by the initial ejection velocity field from the disruption of the parent body and by the Yarkovsky effect. We have demonstrated, following the work of Vokrouhlický et al. (2006a), that the functional form of the spread of a family created entirely initial ejection field of fragments from their parent body's disruption (i.e. with  $C_{\text{YE}} = 0$  in equation 10), and the spread caused by the Yarkovsky effect (i.e. with  $C_{\text{EV}} = 0$  and  $C_{\text{YE}} > 0$  in equation 10) are functionally equivalent.

We have measured the V-shapes of 11 young ( $< 100$  Myr old) asteroid families located within the inner, central and outer MB and we have found that all of them  $\alpha \simeq 1.0$ . We associate this value of  $\alpha$  to the initial velocity V-shape's  $\alpha_{\text{EV}}$ , concluding that the initial ejection velocity is proportional to  $\frac{1}{D}$ , as was already assumed, because these families are too young to have been substantially modified by the Yarkovsky effect. Our  $\alpha$  measurements were repeated for each of the 11 families using the Monte Carlo scheme described in Section 2.3.2 and found that the  $1\sigma$  uncertainty of the trial  $\alpha$  measurements were within  $\sim 5$  per cent of the mean trial  $\alpha$  value for most of the families. Some of the families such as the Aeolia, Clarissa, Brangane, Iannini and Koronis(2) families had slightly skewed trial  $\alpha$  distributions in the positive and negative directions.

The average value of  $\alpha$  of the 11 family V-shapes in this study is  $0.97 \pm 0.02$ , or within in the Student's t-distribution 99.8 per cent confidence interval 0.95–0.99. Additionally, our average measurement of  $\alpha \simeq 1.0$  confirms the results of laboratory and numerical experiments of asteroid disruption events showing  $\alpha_{\text{EV}} \simeq 1.0$  (Fujiwara et al. 1989; Michel et al. 2001; Nesvorný & Bottke 2004). Other studies focusing on modelling the observed distribution of family fragments with disruption simulations such as for the Karin family (Nesvorný et al. 2002, 2006) or the  $i$  distribution of family such as for the Koronis family (Carruba et al. 2016a) also show that  $\alpha \simeq 1.0$ .

The  $\alpha$  determination technique in Section 2.1 can be used to determine whether or not an asteroid family V-shape is young enough

**Table 1.** Description of variables in order of appearance.

Variable	Description
$D$	Asteroid diameter in km
$a$	Semimajor axis in au.
$e$	Eccentricity.
$i$	Inclination in degrees.
$D_r$	Reciprocal of the diameter, $\frac{1}{D}$ in $\text{km}^{-1}$ .
$a_c$	The location of the V-shape center in au.
$n$	Mean motion in $\frac{\text{rad}}{\text{s}}$ .
$V_{\text{ev}}$	Ejection velocity in $\frac{\text{m}}{\text{s}}$ .
$\alpha_{\text{EV}}$	The $\alpha$ of an initial velocity V-shape.
$p_V$	Visual albedo.
$C$	Total V-shape width in au.
$\alpha$	$\alpha$ defined by equation (6).
$N_{\text{out}}$	Number density of objects between the nominal and outer V-shapes.
$N_{\text{in}}$	Number density of objects between the nominal and inner V-shapes.
$dC$	Difference in $C$ between the nominal and outer/inner V-shapes.
$H$	Absolute magnitude.
$C_{\text{YE}}$	V-shape width due to Yarkovsky spreading of fragments in au.
$C_{\text{EV}}$	V-shape width due to the initial ejection of fragments in au.
$N$	Number of family members used with the V-shape technique.
$\alpha_{\text{YE}}$	The $\alpha$ of a Yarkovsky V-shape.
$\rho$	Asteroid density in $\text{g cm}^{-3}$ .
$A$	Bond albedo.
$\theta$	Asteroid obliquity.

**Table 2.** The measured value of  $\alpha$  of young asteroid families: Asteroid family fragment taxonomies are taken from Nesvorný et al. (2003), Willman et al. (2008), Harris et al. (2009), Molnar & Haegert (2009), Novaković, Tsiganis & Knežević (2010), Spoto et al. (2015) and Nesvorný et al. (2015). Diameters for the parent body,  $D_{\text{pb}}$ , were taken from the means of asteroid family parent bodies in Brož et al. (2013) and Durda et al. (2007) if  $D_{\text{pb}}$  was available from both sources.  $D_{\text{pb}}$  for Iannini was taken from Nesvorný et al. (2003). The  $D_{\text{pb}}$  of the Koronis(2) and Veritas families were determined with techniques from Nesvorný et al. (2015) to estimate parent body size.  $p_V$  values of asteroid family members are taken from Masiero et al. (2013) and Spoto et al. (2015).  $N$  is the number of family fragments used in the determination of a family's V-shape.  $D_s$  and  $D_l$  are the boundaries of the  $D$  of the smallest and largest fragments used to measure an asteroid family V-shape's  $\alpha$ .

Designation	Tax.	$D_{\text{pb}}$ (km)	$C_{\text{EV}}$ ( $10^{-6}$ au)	$C$ ( $10^{-6}$ au)	$N$	$a_c$ (au)	$\alpha$	$p_V$	$D_s-D_l$ (km)
1993 FY <sub>12</sub>	S	15	7.5	8.1	87	2.847	$1.03 \pm 0.11$	$0.17 \pm 0.05$	2.0–4.7
Aeolia	X	35	7.7	3.6	225	2.7415	$1.0 \pm 0.07$	$0.11 \pm 0.03$	1.3–3.3
Brangane	S	42	7.2	3.9	171	2.584	$0.95 \pm 0.04$	$0.1 \pm 0.03$	1.5–7.5
Brasilia	X	34	13.4	15.2	548	2.855	$1.0 \pm 0.1$	$0.24 \pm 0.07$	1.0–7.0
Clarissa	C	39	3.9	3.8	179	2.404	$0.95 \pm 0.04$	$0.06 \pm 0.02$	1.1–8.9
Iannini	S	14*	5.6	2.2	129	2.644	$0.97 \pm 0.07$	$0.30 \pm 0.10$	0.9–3.7
Karin	S	40	12.1	3.1	429	2.865	$0.97 \pm 0.05$	$0.21 \pm 0.06$	1.8–10.3
König	C	37	3.3	4.4	315	2.574	$0.91 \pm 0.03$	$0.06 \pm 0.01$	1.4–5.5
Koronis(2)	S	58	11.1	2.3	235	2.869	$1.09 \pm 0.05$	$0.14 \pm 0.04$	1.1–3.6
Theobalda	C	97	10.7	9.0	349	3.179	$0.95 \pm 0.04$	$0.06 \pm 0.02$	2.1–15.2
Veritas	C	124*	21.0	12.2	1135	3.168	$1.01 \pm 0.04$	$0.07 \pm 0.02$	3.2–31.7

to not have been significantly altered by the Yarkovsky effect. There is indication that the value of  $\alpha$  for Yarkovsky V-shapes, or  $\alpha_{\text{YE}}$ , for older families whose fragments have been significantly modified by the Yarkovsky effect as described by equation (6) is between 0.7 and 0.9 due to possible thermal inertia dependence on asteroid size and its effect on the Yarkovsky drift rate as a function of asteroid size (Delbo et al. 2007, 2015; Bolin et al. 2017). Asteroid family V-shapes that have been significantly affected by the Yarkovsky effect will have  $\alpha$  values as described in equation (6) that are closer to inside the range of 0.7–0.9.

In fact, the ages of some of these families have been determined by the use of alternative methods such as backwards integrating the orbits of selected bodies in the families or by modelling the diffusion of fragments caused by chaos (Nesvorný et al. 2002, 2003;

Tsiganis, Knežević & Varvoglis 2007; Novaković et al. 2010; Carruba, Vokrouhlický & Nesvorný 2017). By these methods, Iannini, Karin, Theobalda and Veritas all have ages between 5 and 9 Myr independently ruling out significant modification of the Yarkovsky effect on their fragments'  $a$ .

We demonstrate that V-shapes in  $a$  versus  $D_r$  space created purely by the initial ejection of fragments can be separated from those that are created by a combination of the initial ejection velocity of fragments and the Yarkovsky effect using the measurement of  $\alpha$  as generically described for asteroid family V-shapes by equation (6). We assume that the value of  $\alpha_{\text{YE}} \sim 0.8$  as indicated by Bolin et al. (2017) for very old families which have lost memory of their initial dispersion. We find that the time-scale for  $\alpha$  to reach  $\alpha_{\text{YE}}$  as a result of the modification of fragments'  $a$  by the Yarkovsky effect to is on



the order of  $\sim 20$  Myr as seen in Fig. 7. Interestingly, the backwards integration technique is unable to identify families and determine accurate family ages for families older than  $\sim 20$  Myr (Nesvorný et al. 2003; Radović 2017).

The V-shape identification method measuring family V-shapes'  $\alpha$  provides a way to distinguish whether a family V-shape is caused by a combination of the initial ejection of fragments and the Yarkovsky effect, or only due to initial ejection of fragments. The measurement of asteroid family V-shape provides an additional independent evidence of the subsequent orbital evolution of asteroid family fragment due to the Yarkovsky effect after the initial placement of fragments due to the initial ejection of fragments because asteroid families old enough to have their fragments' orbits modified by the Yarkovsky will have an  $\alpha < 1.0$  compared to the case where asteroid family fragments' orbits remain unmodified after their parent's disruption where their family V-shape would have  $\alpha = 1.0$ .

It may be possible to independently constrain the degree to which an asteroid family fragments' spread in  $a$  has been modified in part by the Yarkovsky effect relative to the spread caused by the initial ejection of fragments. Family V-shapes with a higher proportion of  $C_{EV}$  relative to  $C_{YE}$  in their total value of  $C$  may have a higher  $\alpha < 1$  compared to family V-shapes with a higher proportion of  $C_{YE}$  relative to  $C_{EV}$  in their total value of  $C$ . Distinguishing the families with higher proportion of  $C_{EV}$  compared to  $C_{YE}$  requires measurements of  $\alpha$  with small uncertainties. Using methods of removing outliers by colours and other physical data such as the method of Radović (2017) may improve the precision of  $\alpha$  measurements independently of other methods such as the V-shape criterion of Nesvorný et al. (2015), which may bias the measurement of a V-shape's  $\alpha$  towards the assumption of  $\alpha$  used in the V-shape criterion before the actual measurement of  $\alpha$  is made. Understanding more about the evolution of family fragments'  $a$  and its effect on the measurement of  $\alpha$  may provide an independent constraint on a family's age provided the physical properties of the asteroid family fragments are known such as density and surface thermal conductivity.

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## APPENDIX A:

### A1 1993 FY<sub>12</sub>

The 1993 FY<sub>12</sub> asteroid family located in the outer Main Belt was first identified by Nesvorný (2010) and consists of mostly S-type asteroids (Spoto et al. 2015). The age of the family is roughly estimated to be <200 Myr by Brož et al. (2013) and  $83 \pm 28$  Myr by Spoto et al. (2015). It should be noted that ages of asteroid families by Spoto et al. (2015) are upper limits to the family age because they are computed with the assumption that  $C_{EV} \simeq 0$ . The V-shape identification technique was applied to 87 asteroids belonging to the 1993 FY<sub>12</sub> asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function  $\delta(a_j - a)$  because the inner half of the family V-shape for 1993 FY<sub>12</sub> is more densely populated than the outer V-shape half possibly due to lack of completeness of 1993 FY<sub>12</sub> family members as seen in the bottom panel of Fig. A1. The

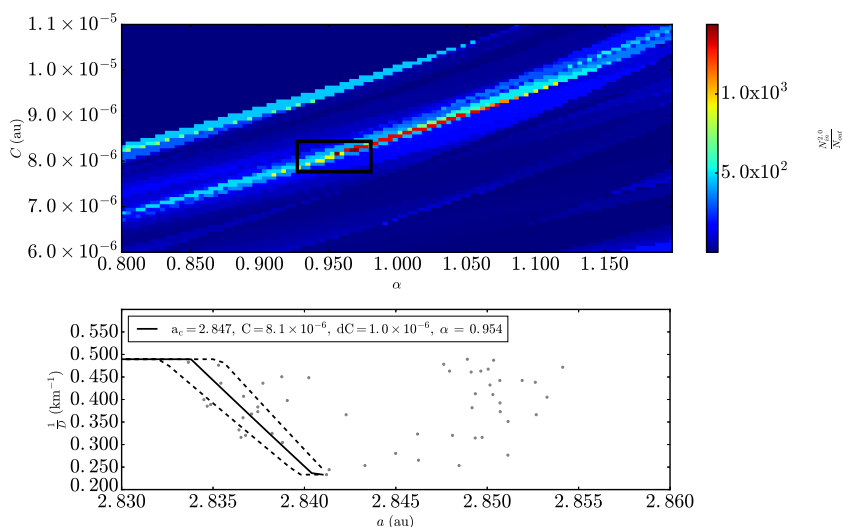
interval  $[0.21, 0.49]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  to avoid the potential distortion of the family V-shape because of smaller fragments interacting with the 5:2 MMR at 2.81 au. Equation (6) is truncated to 0.21 for  $D_r < 0.21$  and to 0.49 for  $D_r > 0.49$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.184$  typical for members of the 1993 FY<sub>12</sub> family (Spoto et al. 2015).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.847 \text{ au}, 8.1 \times 10^{-6} \text{ au}, \sim 0.95)$  as seen in the top panel of Fig. A1 is located in the range of  $2.83 < a < 2.86 \text{ au}$ ,  $6.0 \times 10^{-6} < C < 1.1 \times 10^{-5} \text{ au}$  and  $0.8 < \alpha < 1.2$ . A  $dC = 1.0 \times 10^{-6} \text{ au}$  was used. The technique was repeated with the 1993 FY<sub>12</sub> family defined by Spoto et al. (2015) resulting in identical results.

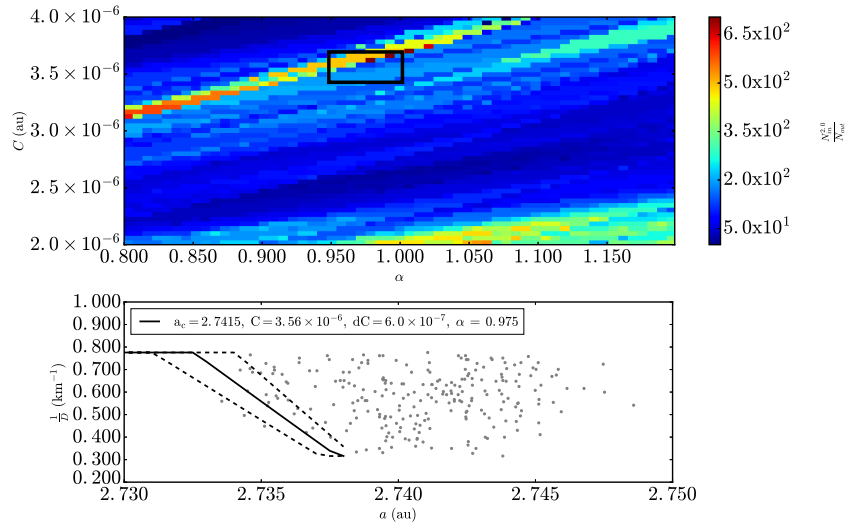
The V-shape identification technique was repeated in  $\sim 4900$  Monte Carlo runs where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be the average value of  $p_V$  for family fragments in the 1993 FY<sub>12</sub> family fragments of 0.18 with an uncertainty of 0.04 (Spoto et al. 2015). The value of  $\alpha$  in  $\sim 4900$  Monte Carlo trials ranges between  $0.5 \lesssim \alpha \lesssim 1.5$  and is, on average,  $\sim 1.0$  with an RMS uncertainty of 0.11 as seen in Fig. 6. The large RMS uncertainty of 0.11 is due to the low number of asteroids used to measure the family V-shape's  $\alpha$ . The value of  $\mu_\alpha \simeq 1.0$  and a similar value of  $C = 8.1 \times 10^{-6}$  compared to  $C_{EV} = 7.50 \times 10^{-6}$  calculated using equation (7) assuming  $V_{EV} = 15 \text{ m s}^{-1}$  from Brož et al. (2013) suggest that the spread of fragments in the 1993 FY<sub>12</sub> family in  $a$  versus  $D_r$  space is almost entirely due to the ejection velocity of the fragments with minimal modification in  $a$  due to the Yarkovsky effect.

### A2 Aeolia

The X-type Aeolia asteroid family located in the outer Main Belt was first identified by Nesvorný (2010). The age of the family is estimated to be only <100 Myr by Brož et al. (2013) and  $100 \pm 18$  Myr by Spoto et al. (2015). The V-shape identification technique was applied to 225 asteroids belonging to the Aeolia asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function



**Figure A1.** The same as Fig. 1 for the 1993 FY<sub>12</sub> asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3} \text{ au}$  and  $\Delta C$  is equal to  $1.5 \times 10^{-7} \text{ au}$ . Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.17$ .  $a_c = 2.847 \text{ au}$  and  $dC = 8.0 \times 10^{-7} \text{ au}$ .



**Figure A2.** The same as Fig. 1 for Aeolia asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3}$  au and  $\Delta C$  is equal to  $8.0 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.11$ ,  $a_c = 2.742$  au and  $dC = 3.56 \times 10^{-6}$  au.

$\delta(a_j - a)$  because the outer half of the family V shape for Aeolia has a less defined border than the inner V-shape half due to possible interlopers as seen in the bottom panel of Fig. A2. The V-shape criterion of Nesvorný et al. (2015) was not used to remove potential interlopers because it assumes a functional form of  $\alpha = 1.0$  in equation (6), which would result in artificially trimming and biasing the family V-shape towards having an  $\alpha = 1.0$ . The interval  $[0.31, 0.78]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to remove at higher  $D_r$  values on the inner edge of the family V-shape in the application of the V-shape identification technique. Equation (6) is truncated to 0.31 for  $D_r < 0.31$  and to 0.78 for  $D_r > 0.78$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.11$  typical for members of the Aeolia family (Spoto et al. 2015).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.742 \text{ au}, 3.56 \times 10^{-6} \text{ au}, \sim 0.975)$  as seen in the top panel of Fig. A2 is located in the range of  $2.73 < a < 2.75$  au,  $2.0 \times 10^{-6} < C < 4.0 \times 10^{-6}$  au and  $0.8 < \alpha < 1.2$ . A  $dC = 6.0 \times 10^{-7}$  au was used. The technique was repeated with the Aeolia family defined by Spoto et al. (2015) resulting in similar results. In addition, the Aeolia family is noted by Spoto et al. (2015) for having an asymmetrical V-shape where the outer half the V-shape has a steeper slope. The  $a_c, C, \alpha$  technique was repeated using  $[a_c, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  resulting in a peak located at  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.742 \text{ au}, 2.46 \times 10^{-6} \text{ au}, \sim 0.968)$ . A lower value of  $C$  of  $2.46 \times 10^{-6}$  for the outer V-shape half compared to the value of  $C$  of  $3.56 \times 10^{-6}$  is in agreement with Spoto et al. (2015) for the outer V-shape half of the Aeolia family having a smaller value of  $C$ .

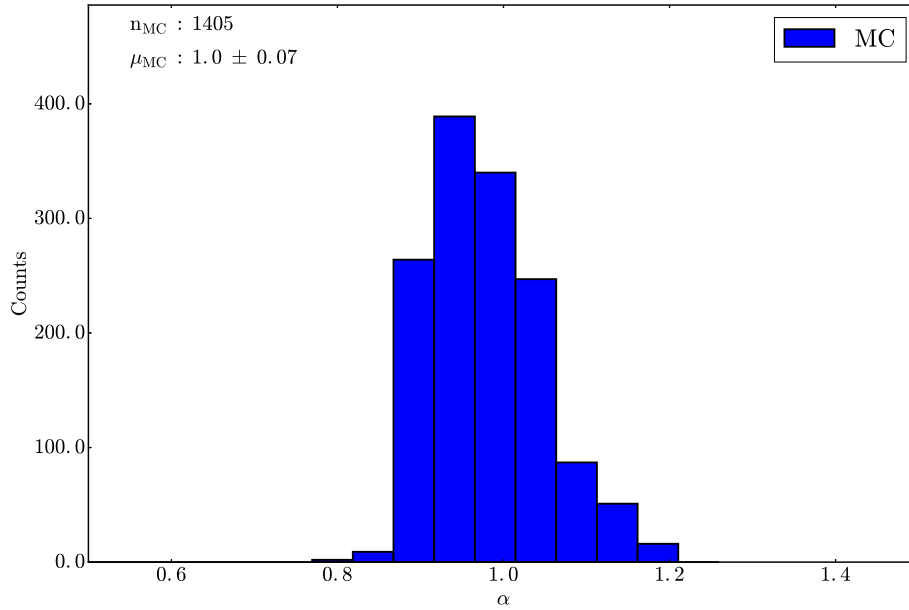
A total of 1405 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.11, on average, with an uncertainty of 0.03 (Spoto et al. 2015). The value of  $\alpha$  in 1405 Monte Carlo trials is, on average,  $\sim 1.0$  with an RMS uncertainty of 0.07 as seen in Fig. A3. The large RMS uncertainty of 0.07 is due to the low number of asteroids used to measure the family V-shape's  $\alpha$  on the inner half of the family V shape. The  $\alpha$  distribution of Monte Carlo runs is slightly positively skewed such that the most probable value is slightly lower than the mean of  $\sim 1.0$ . The

value of  $\mu_\alpha \simeq 1.0$  and a smaller value of  $C = 3.6 \times 10^{-6}$  au compared to  $C_{EV} = 7.7 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = 22 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 35 \text{ km}$  (Durda et al. 2007; Brož et al. 2013) and  $\rho = 2.7 \text{ g cm}^{-3}$  typical for X-type asteroids (Carry 2012) suggests that the spread of fragments in the Aeolia family in  $a$  versus  $D_r$  space is due to the ejection velocity of the fragments with minimal modification in  $a$  due to the Yarkovsky effect.

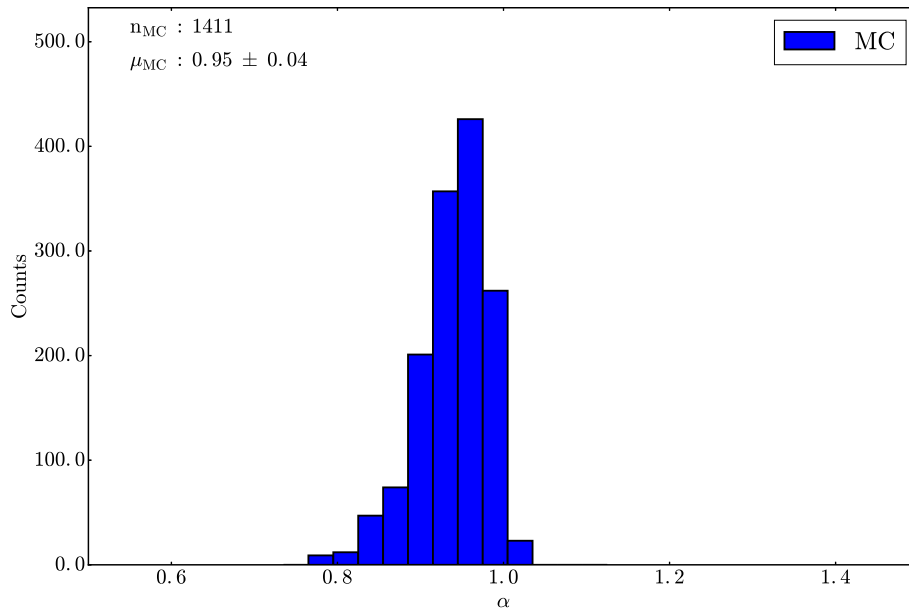
### A3 Brangane

The S-type Brangane asteroid family located in the central Main Belt was first identified by Nesvorný (2010). The age of the family is estimated to be  $50 \pm 40 \text{ Myr}$  by Brož et al. (2013) with a similar estimate in Spoto et al. (2015). The V-shape identification technique was applied to 171 asteroids belonging to the Brangane asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function  $\delta(a_j - a)$  because the outer half of the family V-shape for Brangane has a fewer asteroids in  $a$  versus  $D_r$  space on its outer V-shape half as seen panel of Fig. 3. The interval  $[0.19, 0.92]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to cover the range of the entire inner half of the Brangane family's V-shape. Equation (6) is truncated to 0.19 for  $D_r < 0.19$  and to 0.92 for  $D_r > 0.92$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.10$  typical for members of the Brangane family (Masiero et al. 2013; Spoto et al. 2015).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.584 \text{ au}, 3.89 \times 10^{-6} \text{ au}, \sim 0.95)$  as seen in the top panel of Fig. 3 is located in the range of  $2.57 < a < 2.60$  au,  $3.5 \times 10^{-6} < C < 5.5 \times 10^{-6}$  au and  $0.7 < \alpha < 1.3$ . A  $dC = 1.0 \times 10^{-6}$  au was used. The technique was repeated with the Brangane family defined by Spoto et al. (2015) resulting in similar results as seen in Fig. 4. The  $a_c, C, \alpha$  technique was repeated using  $[a_c, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  resulting in a peak located at  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.5835 \text{ au}, 3.05 \times 10^{-6} \text{ au}, \sim 0.985)$ . A lower value of  $C$  of  $3.05 \times 10^{-6}$  for the outer V-shape half compared to the value of  $C$  of  $3.83 \times 10^{-6}$  is in agreement with Spoto et al. (2015) for the outer V-shape half of the Brangane family having a smaller value of  $C$ .



**Figure A3.** The same as Fig. 5 with  $\sim 1930$  trials repeating the V-shape technique for the Aeolia family. The mean of the distribution is centred at  $\alpha = 1.0 \pm 0.07$  and the bin size in the histogram is 0.05.



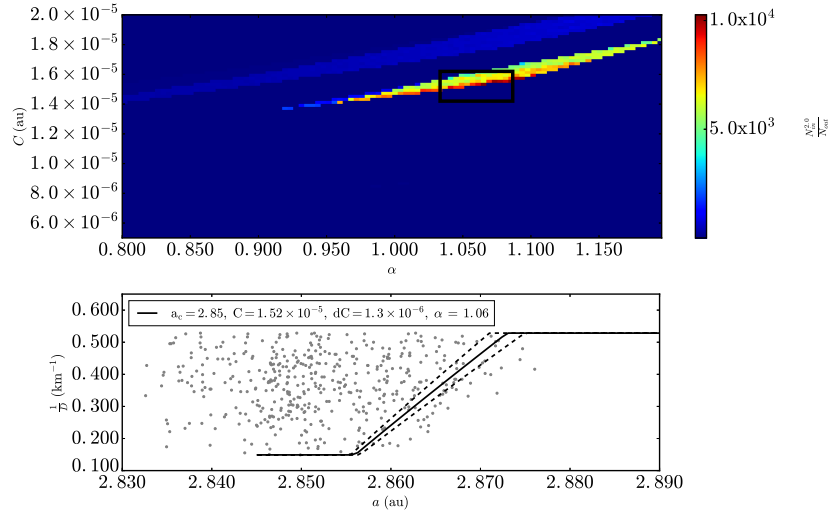
**Figure A4.** The same as Fig. 5 with  $\sim 1400$  trials repeating the V-shape technique for the Brangane family. The mean of the distribution is centred at  $\alpha = 0.95 \pm 0.04$  and the bin size in the histogram is 0.03.

Approximately 1400 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.10 with an uncertainty of 0.03 (Masiero et al. 2013). The value of  $\alpha$  in 1400 Monte Carlo trials is, on average,  $\sim 0.95$  with an RMS uncertainty of 0.04 as seen in Fig. A4. The  $\alpha$  distribution of Monte Carlo runs is slightly negatively skewed such that the most probable value is slightly higher than the mean of 0.95. The value of  $\mu_\alpha \lesssim 1.0$  and a smaller value of  $C = 3.90 \times 10^{-6}$  au compared to  $C_{EV} = 7.24 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = 23 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,

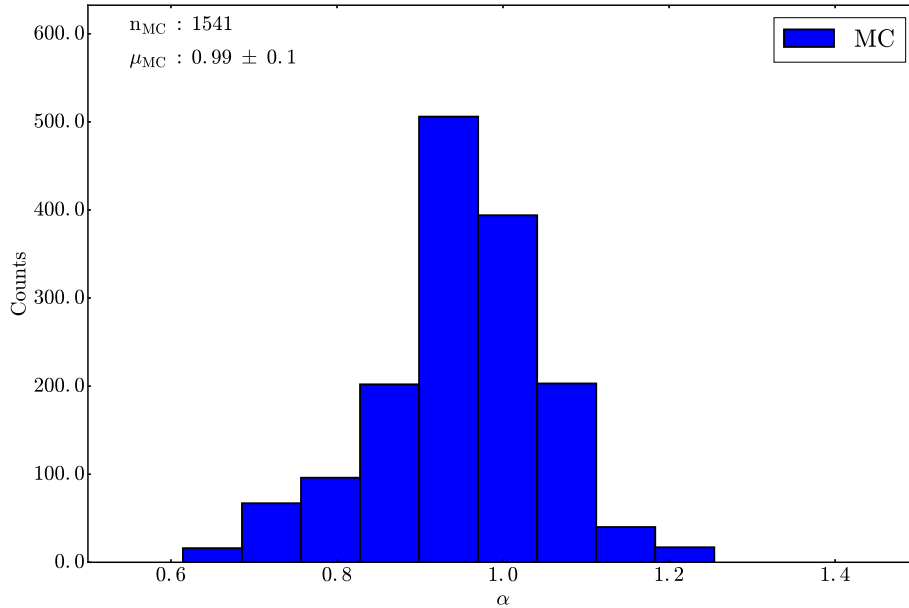
$D_{pb} = 42 \text{ km}$  (Brož et al. 2013) and  $\rho = 2.3 \text{ g cm}^{-3}$  typical for S-type asteroids (Carry 2012) suggests that the spread of fragments in the Brangane family in  $a$  versus  $D_r$  space is mostly due to the ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect.

#### A4 Brasilia

The X-type fragment Brasilia asteroid family first identified by Zappalà et al. located in the outer Main Belt. The M/N Solar system dust band was later attributed to the formation of the family with the asteroid 293 Brasilia being an interloper in its own family



**Figure A5.** The same as Fig. 1 for Brasilia asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $4.0 \times 10^{-3}$  au and  $\Delta C$  is equal to  $6.5 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.24$ ,  $a_c = 2.855$  au and  $dC = 1.3 \times 10^{-6}$  au.



**Figure A6.** The same as Fig. 5 with  $\sim 1500$  trials repeating the V-shape technique for the Brasilia family. The mean of the distribution is centred at  $\alpha \simeq 1.0 \pm 0.10$  and the bin size in the histogram is 0.07.

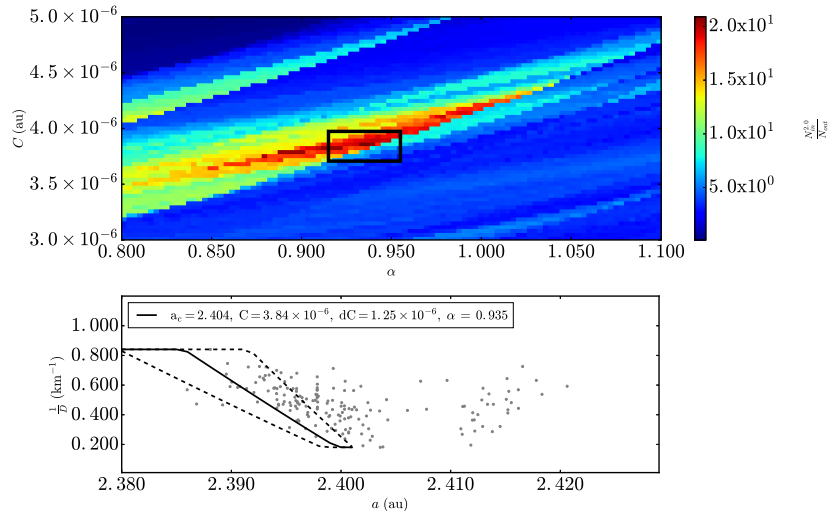
(Nesvorný et al. 2003; Brož et al. 2013). The asteroid 1521 Sejna-joki is more likely to be the largest asteroid family member in the Brasilia asteroid family, but we will use Brasilia as the name for the asteroid family. The age of the Brasilia family is estimated to be  $50 \pm 40$  Myr by Brož et al. (2013). The V-shape identification technique was applied to 584 asteroids belonging to the Brasilia asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $[a_c, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  because the inner half of the family V-shape for Brasilia is clipped due to the presence of the 5:2 MMR at 2.81 au as seen in the bottom panel of Fig. A5. The interval  $[0.13, 0.53]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to cover the majority of the range of the outer V-shape half while excluding possible interlopers at larger values of  $D_r$ . Equation (6) is truncated to 0.13 for  $D_r < 0.13$  and to 0.53 for  $D_r > 0.53$ . Asteroid  $H$  values were

converted into  $D$  using equation (11) using the value of  $p_V = 0.24$  typical for members of the Brasilia family (Masiero et al. 2013; Spoto et al. 2015).

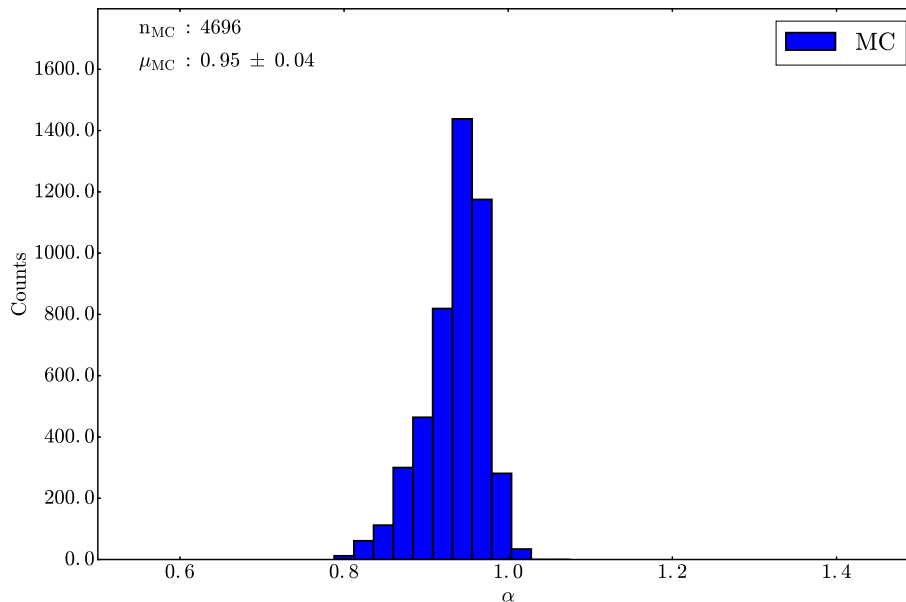
The peak in  $\frac{N_{in}^2}{N_{out}^2}$  at  $(a_c, C, \alpha) = (2.85 \text{ au}, 1.52 \times 10^{-6} \text{ au}, \sim 1.06)$  as seen in the top panel of Fig. A5 is located in the range of  $2.83 < a < 2.89$  au,  $5 \times 10^{-6} < C < 2.0 \times 10^{-5}$  au and  $0.8 < \alpha < 1.2$ . A  $dC = 1.3 \times 10^{-6}$  au was used. The technique was repeated with the Brasilia family defined by Spoto et al. (2015) resulting in similar results.

Approximately 1500 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.24 with an uncertainty of 0.06 (Masiero et al. 2013). The value of  $\alpha$  in  $\sim 1500$  Monte Carlo trials is, on average,  $\sim 0.99$  with an RMS uncertainty





**Figure A7.** The same as Fig. 1 for Clarissa asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $3.0 \times 10^{-3}$  au and  $\Delta C$  is equal to  $3.5 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.06$ ,  $a_c = 2.404$  au and  $dC = 1.25 \times 10^{-6}$  au.



**Figure A8.** The same as Fig. 5 with  $\sim 4,700$  trials repeating the V-shape technique for the Clarissa family. The mean of the distribution is centred at  $\alpha = 0.95 \pm 0.04$  and the bin size in the histogram is 0.07.

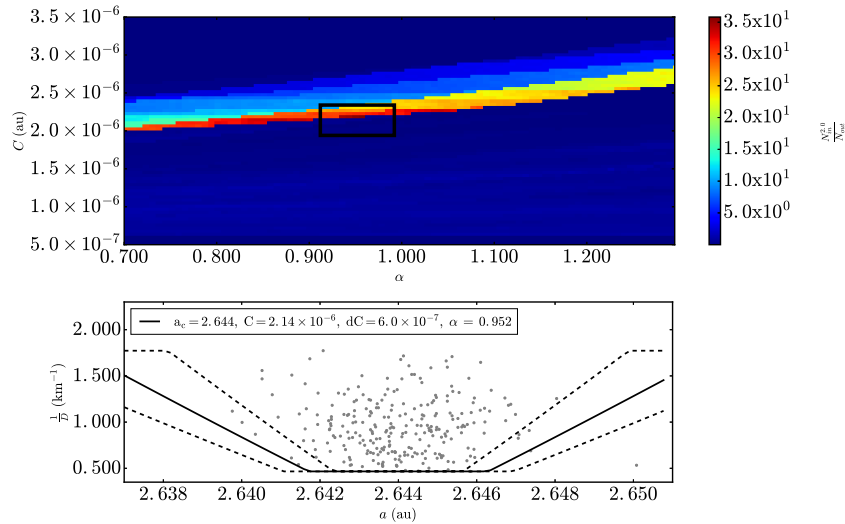
of 0.1 as seen in Fig. A6. The value of  $\mu_\alpha \sim 1.0$  and a similar value of  $C = 1.5 \times 10^{-5}$  au compared to  $C_{EV} = 1.3 \times 10^{-5}$  au calculated from equation (7) assuming  $V_{EV} = 22 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 34$  km (Brož et al. 2013) and  $\rho = 2.7 \text{ g cm}^{-3}$  typical for X-type asteroids (Carr 2012) suggests that the spread of fragments in the Brasilia family in  $a$  versus  $D_r$  space is mostly due to the ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect.

### A5 Clarissa

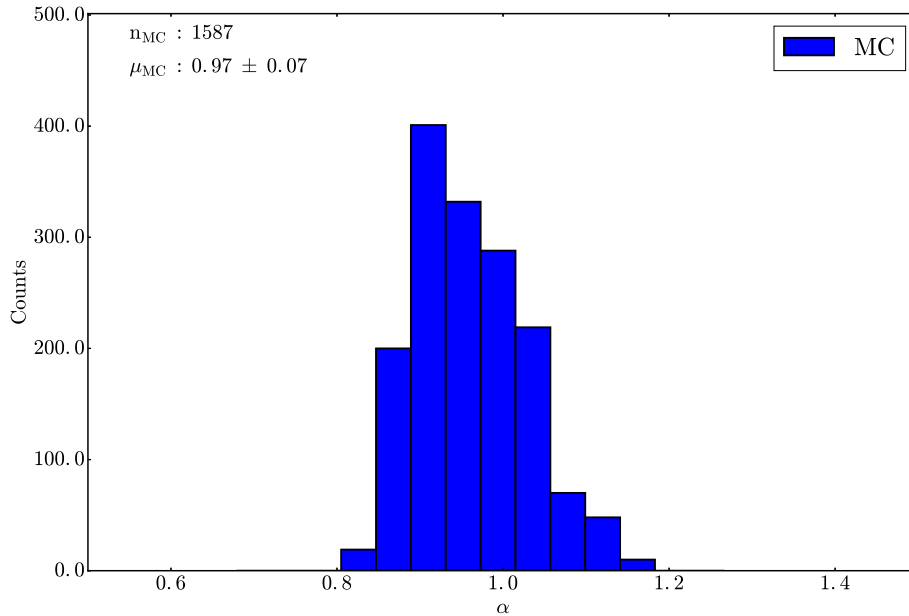
The C complex Clarissa asteroid family located in the inner Main Belt was first identified by Nesvorný (2010). The age of the family is estimated to be  $< 100$  Myr by Brož et al. (2013). The V-shape

identification technique was applied to 179 asteroids belonging to the Clarissa asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function  $\delta(a_j - a)$  because the outer half of the family V-shape for Clarissa has a fewer asteroids in  $a$  versus  $D_r$  space on its outer V-shape half as seen panel of Fig. A7. The interval  $[0.11, 0.91]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to cover the full range of fragments in the Clarissa family inner half V-shape. Equation (6) is truncated to 0.11 for  $D_r < 0.11$  and to 0.91 for  $D_r > 0.91$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.06$  typical for members of the Clarissa family (Masiero et al. 2013).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.404 \text{ au}, 3.84 \times 10^{-6} \text{ au}, \sim 0.94)$  as seen in the top panel of Fig. A7 is located in the range of  $2.38 < a < 2.43$  au,  $3.0 \times 10^{-6} < C < 5.0 \times 10^{-6}$  au and  $0.8 < \alpha < 1.1$ . A  $dC = 1.25 \times 10^{-6}$  au was used.



**Figure A9.** The same as Fig. 1 for Iannini asteroid family data from Nesvorný et al. (2015). (Top panel)  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3}$  au and  $\Delta C$ , is equal to  $4.0 \times 10^{-8}$  au. (Bottom panel)  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.36$ ,  $a_c = 2.644$  au and  $dC = 6.0 \times 10^{-7}$  au.



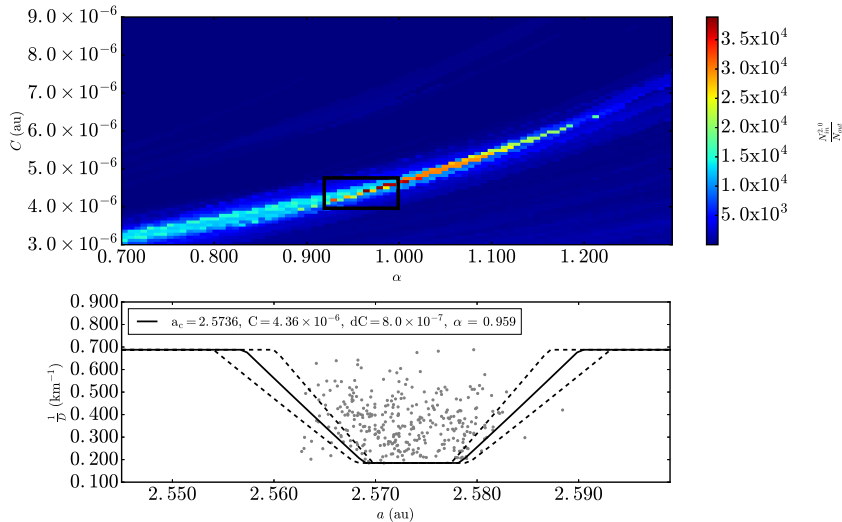
**Figure A10.** The same as Fig. 5 with  $\sim 1600$  trials repeating the V-shape technique for the Iannini family. The mean of the distribution is centred at  $\alpha = 0.97 \pm 0.07$  and the bin size in the histogram is 0.042.

Approximately 4700 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.06 with an uncertainty of 0.02 (Masiero et al. 2013). The value of  $\alpha$  in  $\sim 4700$  Monte Carlo trials is, on average,  $\sim 0.95$  with an RMS uncertainty of 0.04 as seen in Fig. A8. The  $\alpha$  distribution of Monte Carlo runs is slightly negatively skewed such that the most probable value is slightly higher than the mean of 0.95. The value of  $\mu_\alpha \lesssim 1.0$  and a similar value of  $C = 3.84 \times 10^{-6}$  au compared to  $C_{EV} = 3.91 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = 17 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 39 \text{ km}$  (Brož et al. 2013) and  $\rho = 1.4 \text{ g cm}^{-3}$  typical for C-type asteroids (Carry 2012) suggests that the spread of fragments in the Clarissa family in  $a$  versus  $D_r$  space is mostly due to the

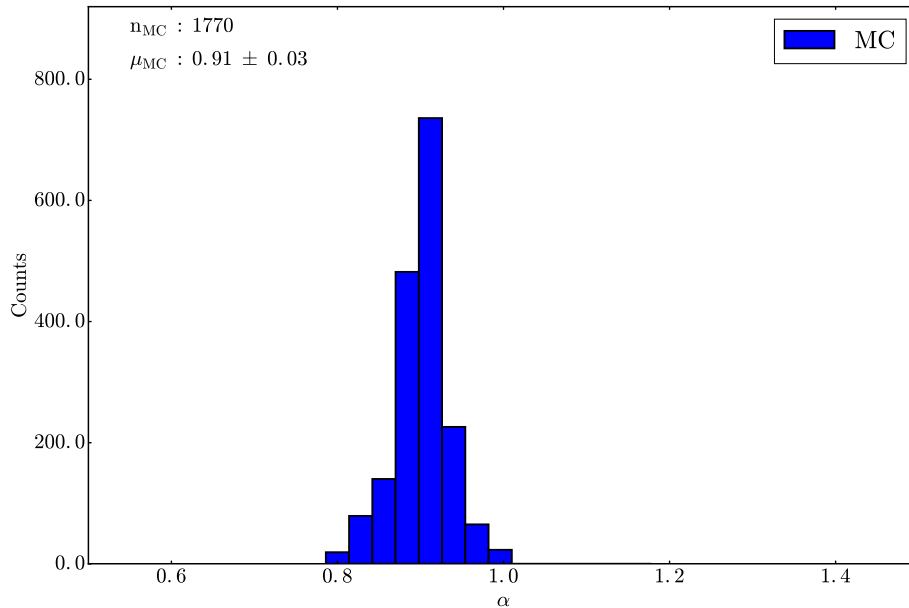
ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect.

## A6 Iannini

The S-type Iannini asteroid family located in the central Main Belt and the presence of the J/K Solar system dust band is attributed to the formation of the family (Nesvorný et al. 2003; Willman et al. 2008). The age of the Iannini family is estimated to be  $5 \pm 5$  Myr by Brož et al. (2013). The V-shape identification technique was applied to 584 asteroids belonging to the Iannini asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  and the interval  $[0.4, 1.5]$  is used for the Dirac delta function  $\delta(D_{r,j} - D_r)$  to contain fragments defining the border of the Iannini family V-shape.



**Figure A11.** The same as Fig. 1 for König asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $7.0 \times 10^{-3}$  au and  $\Delta C$  is equal to  $8.0 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.06$ ,  $a_c = 2.574$  au and  $dC = 8.0 \times 10^{-7}$  au.



**Figure A12.** The same as Fig. 5 with  $\sim 1700$  trials repeating the V-shape technique for the König family. The mean of the distribution is centred at  $\alpha = 0.92 \pm 0.03$  and the bin size in the histogram is 0.03.

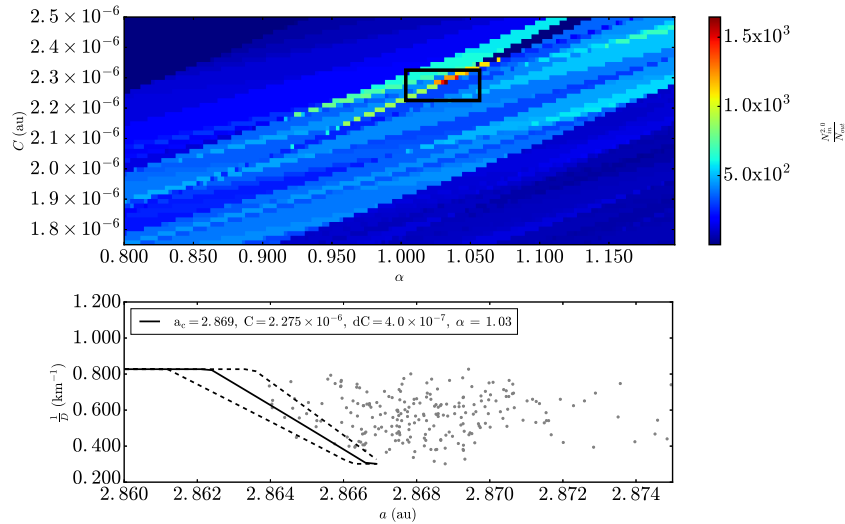
Equation (6) is truncated to 0.4 for  $D_r < 0.4$  and to 1.5 for  $D_r > 1.5$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.32$  typical for members of the Iannini family (Masiero et al. 2013).

The technique was repeated with the Iannini family defined by Spoto et al. (2015). The Iannini family defined by Spoto et al. (2015) differs from the definition of the Iannini family by Nesvorný et al. (2015) by using the asteroid Nele as the largest fragment in the family. If the 17 km diameter asteroid Nele is considered to be the largest remaining fragment of the Iannini family, the family V-shape is slightly asymmetric Spoto et al. (2015). Regardless of the potential asymmetric V-shape of the Iannini/Nele family, the V-shape has a symmetrical shape in both Spoto et al. (2015) and Nesvorný et al. (2015) definitions of the Iannini family in the interval  $0.4 \leq D_r \leq 1.5$ . This interval does not include asteroids Iannini and

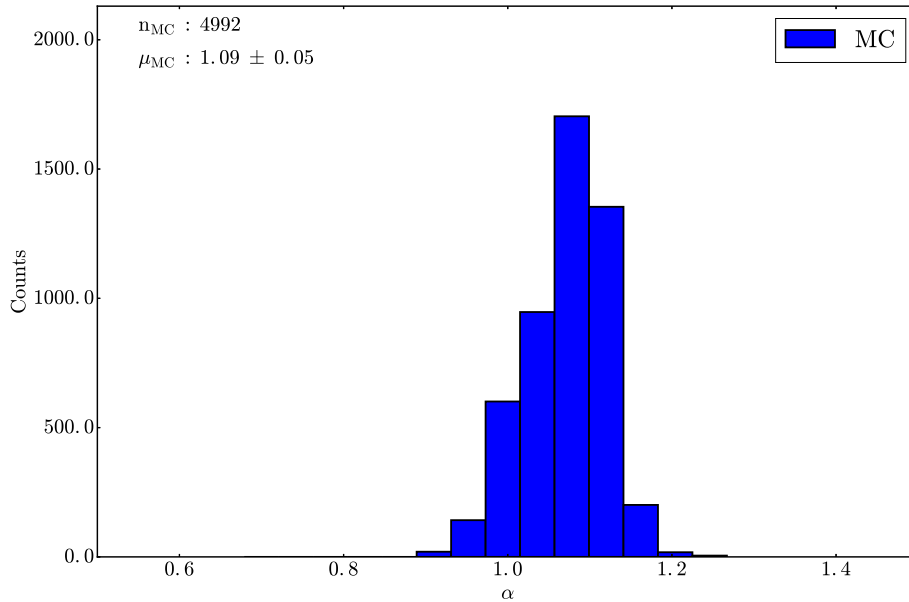
Nele that have  $D_r$  equal to  $0.2 \text{ km}^{-1}$  and  $0.06 \text{ km}^{-1}$ , respectively, and the results of the  $a_c$ ,  $C$  and  $\alpha$  determination technique are similar when applied to both catalogues. An alternative explanation to the membership of Nele to the Iannini family is that it is an interloper because of how far it is offset from the apex of the main family V-shape (Nesvorný et al. 2015).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.64 \text{ au}, 2.1 \times 10^{-6} \text{ au}, \sim 0.95)$  as seen in the top panel of Fig. A9 is located in the range of  $2.637 < a < 2.65 \text{ au}$ ,  $0.5 \times 10^{-6} < C < 3.5 \times 10^{-6} \text{ au}$  and  $0.7 < \alpha < 1.3$ . A  $dC = 6.0 \times 10^{-7}$  au was used. The technique was repeated with the Iannini family defined by Spoto et al. (2015) resulting in similar results.

$\sim 1400$  Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011;



**Figure A13.** The same as Fig. 1 for Koronis(2) asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $2.5 \times 10^{-3}$  au and  $\Delta C$  is equal to  $1.5 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.14$ ,  $a_c = 2.87$  au and  $dC = 4.0 \times 10^{-7}$  au.



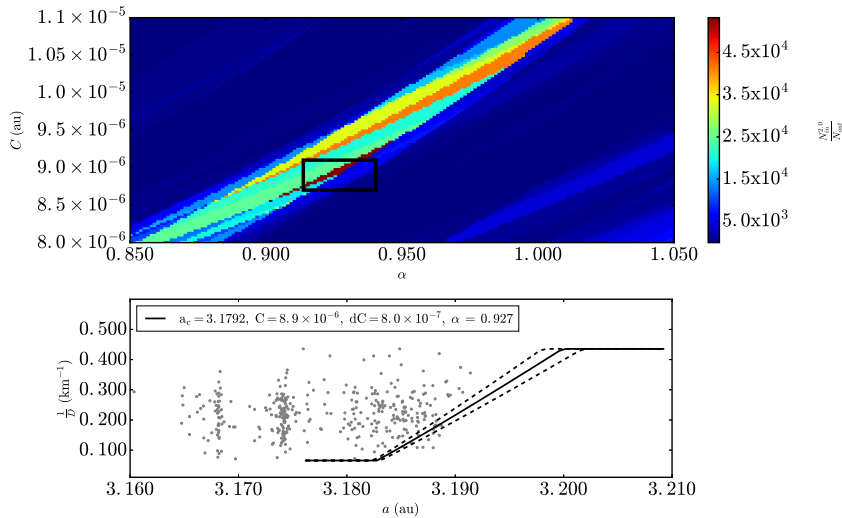
**Figure A14.** The same as Fig. 5 with  $\sim 5,000$  trials repeating the V-shape technique for the Koronis(2) family. The mean of the distribution is centred at  $\alpha = 1.09 \pm 0.05$  and the bin size in the histogram is 0.04.

Vereš et al. 2015) and their  $p_V$  was assumed to be 0.32 with an uncertainty of 0.1 (Masiero et al. 2013). The value of  $\alpha$  in  $\sim 1600$  Monte Carlo trials ranges between  $0.5 \lesssim \alpha \lesssim 1.5$  and is, on average,  $\sim 0.97$  with an RMS uncertainty of 0.07 as seen in Fig. A10. The  $\alpha$  distribution of Monte Carlo runs is slightly positively skewed such that the most probable value is slightly lower than the mean of 0.97. The value of  $\mu_\alpha \sim 1.0$  and a smaller value of  $C = 2.1 \times 10^{-6}$  au compared to  $C_{EV} = 3.3 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = \sim 5.6 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 10 \text{ km}$  (Nesvorný et al. 2015) and  $\rho = 2.3 \text{ g cm}^{-3}$  typical for S-type asteroids (Carry 2012) suggests that the spread of fragments in the Iannini family in  $a$  versus  $D_r$  space is mostly due to the ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect. This conclusion is strengthened if Nele is considered to be the largest remnant asteroid of the Iannini family increasing  $D_{pb}$  to 22 km,

which increases  $C_{EV}$  to  $7.3 \times 10^{-6}$  au, now more than three times larger than the measured value of  $C = 2.1 \times 10^{-6}$  au for the family V-shape.

## A7 Karin

The Karin asteroid family located in the outer Main Belt has an initial ejection velocity field that scales with  $\alpha_{EV} = 1.0$  (Nesvorný et al. 2002, 2006) and is known to contain S-type asteroids (Molnar & Haegert 2009; Harris et al. 2009). The age of the Karin family is estimated to be  $5.75 \pm 0.01 \text{ Myr}$  by Carruba et al. (2016b). The V-shape identification technique was applied to 429 asteroids belonging to the Karin asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  and the interval  $[0.2, 1.1]$  is used for the Dirac delta



**Figure A15.** The same as Fig. 1 for Theobalda asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $1.3 \times 10^{-3}$  au and  $\Delta C$ , is equal to  $3.0 \times 10^{-8}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.06$ ,  $a_c = 3.18$  au and  $dC = 8.0 \times 10^{-7}$  au.

function  $\delta(D_{r,j} - D_r)$  to contain fragments on the border of the Karin family V-shape. Equation (6) is truncated to 0.2 for  $D_r < 0.2$  and to 1.1 for  $D_r > 1.1$  as seen in the bottom panel of Fig. 2. Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.21$  typical for members of the Karin family (Harris et al. 2009).

The peak in  $\frac{N_{in}^2}{N_{out}^2}$  at  $(a_c, C, \alpha) = (2.86 \text{ au}, 3.1 \times 10^{-6} \text{ au}, \sim 1.05)$  as seen in the top panel of Fig. 2 is located in the range of  $2.850 < a < 2.875$  au,  $2.0 \times 10^{-6} < C < 5.5 \times 10^{-6}$  au and  $0.7 < \alpha < 1.3$ . A  $dC = 2.0 \times 10^{-6}$  au was used.

$\sim 1200$  Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.21 with an uncertainty of 0.06 (Harris et al. 2009). The value of  $\alpha$  in  $\sim 1200$  Monte Carlo trials is, on average,  $\sim 0.97$  with an RMS uncertainty of 0.05 as seen in Fig. 5. The value of  $\mu_\alpha \sim 1.0$  and a smaller value of  $C = 3.1 \times 10^{-6}$  au compared to  $C_{EV} = 12.1 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = \sim 22.5 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 40$  km (Durda et al. 2007) and  $\rho = 2.3 \text{ g cm}^{-3}$  typical for S-type asteroids (Carry 2012) suggests that the spread of fragments in the Karin family in  $a$  versus  $D_r$  space is mostly due to the ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect. This is in agreement with the results of Carruba et al. (2016b), which found that the smallest fragments in the Karin family only drifted  $\sim 10^{-3}$  au over the lifetime of the family making only  $\sim 10^{-7}$  au of a difference in the value of the family V-shapes'  $C$  value.

#### A8 König

The König asteroid family located in the central Main Belt its formation is attributed to the G/H Solar system dust band (Nesvorný et al. 2003) and is known to contain C-type asteroids (Brož et al. 2013). The age of the König family is estimated to be between 50 and 100 Myr (Spoto et al. 2015). The V-shape identification technique was applied to 315 asteroids belonging to the König asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, \infty)$  for the Dirac delta function

$\delta(a_j - a)$  and the interval  $[0.18, 0.7]$  is used for the Dirac delta function  $\delta(D_{r,j} - D_r)$  to contain fragments on the border of the König family V-shape as seen in the bottom panel of Fig. A11. Equation (6) is truncated to 0.18 for  $D_r < 0.18$  and to 0.7 for  $D_r > 0.7$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.06$  typical for members of the König family (Spoto et al. 2015).

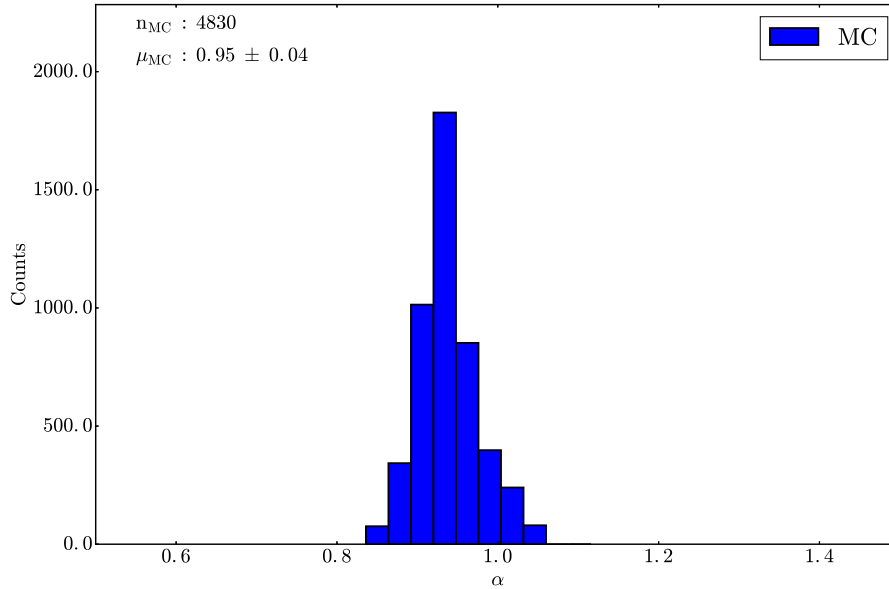
The peak in  $\frac{N_{in}^2}{N_{out}^2}$  at  $(a_c, C, \alpha) = (2.57 \text{ au}, 4.4 \times 10^{-6} \text{ au}, \sim 0.96)$  as seen in the top panel of Fig. A11 is located in the range of  $2.55 < a < 2.60$  au,  $3.0 \times 10^{-6} < C < 9.0 \times 10^{-6}$  au and  $0.7 < \alpha < 1.3$ . A  $dC = 8.0 \times 10^{-7}$  au was used. The technique was repeated with the König family defined by Spoto et al. (2015) resulting in similar results.

$\sim 1600$  Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.06 with an uncertainty of 0.02 (Spoto et al. 2015). The value of  $\alpha$  in  $\sim 1600$  Monte Carlo trials is, on average,  $\sim 0.92$  with an RMS uncertainty of 0.03 as seen in Fig. A12. The value of  $\mu_\alpha \sim 0.92$  and a similar value of  $C = 4.4 \times 10^{-6}$  au compared to  $C_{EV} = 3.3 \times 10^{-6}$  au calculated from equation (7) assuming  $V_{EV} = \sim 14.6 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 33$  km (Brož et al. 2013) and  $\rho = 1.4 \text{ g cm}^{-3}$  typical for C-type asteroids (Carry 2012) suggests that the spread of fragments in the König family in  $a$  versus  $D_r$  space is mostly due to the ejection velocity of the fragments with only moderate modification in  $a$  due to the Yarkovsky effect.

#### A9 Koronis(2)

The S-type Koronis(2) asteroid family located in the outer Main Belt was first identified by Molnar & Haegert (2009). The age of the family is estimated to be only  $15 \pm 5$  Myr by Molnar & Haegert (2009) and Brož et al. (2013). The V-shape identification technique was applied to 235 asteroids belonging to the Koronis(2) asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function  $\delta(a_j - a)$  because of the location of possible interlopers in the outer half of the family V-shape for Koronis(2) (see bottom panel of Fig. A13). The interval  $[0.28, 0.89]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to mitigate the presence of potential





**Figure A16.** The same as Fig. 5 with  $\sim 4800$  trials repeating the V-shape technique for the Theobalda family. The mean of the distribution is centred at  $\alpha = 0.95 \pm 0.04$  and the bin size in the histogram is 0.03.

interlopers in the inner half of the Koronis(2) family V-shape during the application of the V-shape identification technique. Equation (6) is truncated to 0.28 for  $D_r < 0.28$  and to 0.89 for  $D_r > 0.89$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.14$  typical for members of the Koronis(2) family (Masiero et al. 2013).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (2.869 \text{ au}, 2.3 \times 10^{-6} \text{ au}, \sim 1.06)$  as seen in the top panel of Fig. A13 is located in the range of  $2.86 < a < 2.88 \text{ au}$ ,  $1.8 \times 10^{-6} < C < 2.5 \times 10^{-6} \text{ au}$  and  $0.8 < \alpha < 1.2$ . A  $dC = 4.0 \times 10^{-7} \text{ au}$  was used.

$\sim 5000$  Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.14 with an uncertainty of 0.04 (Spoto et al. 2015). The value of  $\alpha$  in  $\sim 5000$  Monte Carlo trials is, on average,  $\sim 1.1$  with an RMS uncertainty of 0.05 as seen in Fig. A14. The  $\alpha$  distribution of Monte Carlo runs is slightly negatively skewed such that the most probable value is slightly higher than the mean of  $\sim 1.1$ . The value of  $\mu_\alpha \simeq 1.1$  and a smaller value of  $C = 2.3 \times 10^{-6} \text{ au}$  compared to  $C_{EV} = 1.1 \times 10^{-5} \text{ au}$  calculated from equation (7) assuming  $V_{EV} \simeq 20 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 35 \text{ km}$  (Nesvorný et al. 2015) and  $\rho = 2.3 \text{ g cm}^{-3}$  typical for S-type asteroids (Carry 2012) suggests that the spread of fragments in the Koronis(2) family in  $a$  versus  $D_r$  space is due almost entirely to the ejection velocity of the fragments.

#### A10 Theobalda

The  $7 \pm 2 \text{ Myr}$  old C-type Theobalda asteroid family located in the outer Main Belt was first identified by Novaković et al. (2010). The V-shape identification technique was applied to 97 asteroids belonging to the Theobalda asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $[a_c, \infty)$  for the Dirac delta function  $\delta(a_j - a)$  because the inner half of the family V-shape for Theobalda is affected by several secular resonances as seen Fig. A15 (Novaković et al. 2010). The interval  $[0.06, 0.47]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used

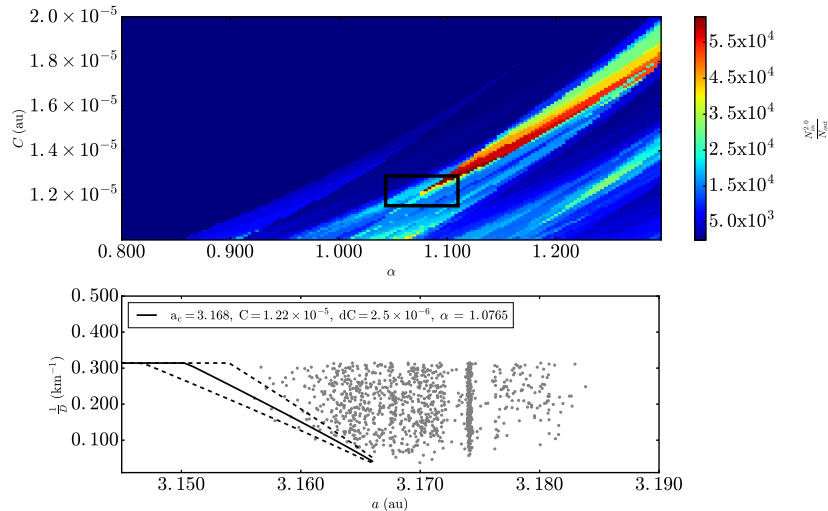
to include asteroids on the outer V-shape border of the Theobalda family. Equation (6) is truncated to 0.06 for  $D_r < 0.06$  and to 0.44 for  $D_r > 0.44$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.06$  typical for members of the Theobalda family (Masiero et al. 2013).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (3.18 \text{ au}, 8.9 \times 10^{-6} \text{ au}, \sim 0.93)$  as seen in the top panel of Fig. A15 is located in the range of  $3.16 < a < 3.21 \text{ au}$ ,  $8.0 \times 10^{-6} < C < 1.2 \times 10^{-5} \text{ au}$  and  $0.85 < \alpha < 1.05$ . A  $dC = 8.0 \times 10^{-7} \text{ au}$  was used.

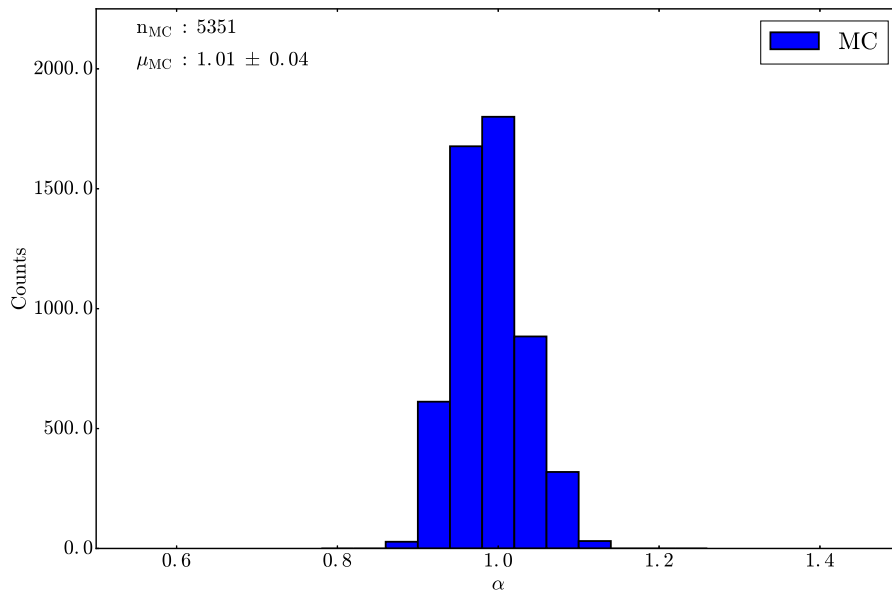
4830 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.06 with an uncertainty of 0.02 (Spoto et al. 2015). The value of  $\alpha$  in 5200 Monte Carlo trials is, on average,  $\sim 0.95$  with an RMS uncertainty of 0.04 as seen in Fig. A16. The value of  $\mu_\alpha \simeq 0.95$  and a smaller value of  $C = 8.9 \times 10^{-6} \text{ au}$  compared to  $C_{EV} = 1.1 \times 10^{-5} \text{ au}$  calculated from equation (7) assuming  $V_{EV} \simeq 20 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 74 \text{ km}$  (Nesvorný et al. 2015) and  $\rho = 1.4 \text{ g cm}^{-3}$  typical for C-type asteroids (Carry 2012) suggests that the spread of fragments in the Theobalda family in  $a$  versus  $D_r$  space is due almost entirely to the ejection velocity of the fragments.

#### A11 Veritas

The Veritas family located in the outer MB and its formation is attributed to the  $\gamma$  Infrared Astronomical Satellite dust band (Nesvorný et al. 2003). The C-type family was first identified by Zappalà et al. (1990) and early studies proposed its age to be  $< 100 \text{ Myr}$  old (Milani & Farinella 1994; Knežević & Pavlović 2002). Subsequent studies further constrained its age to  $8.7 \pm 1.7 \text{ Myr}$  (Nesvorný et al. 2003; Tsiganis et al. 2007; Carruba et al. 2017). The V-shape identification technique was applied to 1135 asteroids belonging to the Veritas asteroid family defined by Nesvorný et al. (2015). Equations (8) and (9) are integrated using the interval  $(-\infty, a_c]$  for the Dirac delta function  $\delta(a_j - a)$  because the outer half and center of the family V-shape for Veritas is affected by



**Figure A17.** The same as Fig. 1 for Karin asteroid family data from Nesvorný et al. (2015). Top panel:  $\Delta\alpha$  is equal to  $3.5 \times 10^{-3}$  au and  $\Delta C$  is equal to  $1.0 \times 10^{-7}$  au. Bottom panel:  $D_r(a, a_c, C \pm dC, p_V, \alpha)$  is plotted with  $p_V = 0.07$ ,  $a_c = 3.17$  au and  $dC = 2.5 \times 10^{-6}$  au.



**Figure A18.** The same as Fig. 5 with  $\sim 5400$  trials repeating the V-shape technique for the Veritas family. The mean of the distribution is centred at  $\alpha = 1.01 \pm 0.04$  and the bin size in the histogram is 0.04.

several secular resonances as seen Fig. A17 (Novaković et al. 2010). The interval  $[0.03, 0.51]$  for the Dirac delta function  $\delta(D_{r,j} - D_r)$  was used to remove interlopers in the inner V-shape half of the Veritas family during the application of the V-shape identification technique. Equation (6) is truncated to 0.03 for  $D_r < 0.03$  and to 0.51 for  $D_r > 0.51$ . Asteroid  $H$  values were converted into  $D$  using equation (11) using the value of  $p_V = 0.07$  typical for members of the Veritas family (Masiero et al. 2013).

The peak in  $\frac{N_{in}^2}{N_{out}}$  at  $(a_c, C, \alpha) = (3.17 \text{ au}, 1.22 \times 10^{-5} \text{ au}, 1.08)$  as seen in the top panel of Fig. A17 is located in the range of  $3.145 < a < 3.190$  au,  $1.0 \times 10^{-5} < C < 2.0 \times 10^{-5}$  au and  $0.80 < \alpha < 1.3$ . A  $dC = 2.5 \times 10^{-6}$  au was used.

Approximately 5350 Monte Carlo runs were completed where  $H$  magnitudes were randomized by the typical magnitude uncertainty of 0.25 for asteroids in the MPC catalogue (Oszkiewicz et al. 2011; Vereš et al. 2015) and their  $p_V$  was assumed to be 0.07 with an

uncertainty of 0.02 (Spoto et al. 2015). The value of  $\alpha$  in  $\sim 5350$  Monte Carlo trials is, on average,  $\sim 1.01$  with an RMS uncertainty of 0.04 as seen in Fig. A18. The value of  $\mu_\alpha \simeq 1.01$  and a smaller value of  $C = 1.22 \times 10^{-5}$  au compared to  $C_{EV} = 2.1 \times 10^{-5}$  au calculated from equation (7) assuming  $V_{EV} \simeq 20 \text{ m s}^{-1}$ , the escape velocity from a parent body with a parent body diameter,  $D_{pb} = 124$  km (Nesvorný et al. 2015) and  $\rho = 1.4 \text{ g cm}^{-3}$  typical for C-type asteroids (Carry 2012) suggests that the spread of fragments in the Veritas family in  $a$  versus  $D_r$  space is due almost entirely to the ejection velocity of the fragments. The  $D_{pb}$  we use may be wrong because it includes asteroid Veritas as a fragment with a diameter of  $\sim 100$  km because Veritas may be an interloper in its own family as indicated by impact modelling (Michel et al. 2011).

This paper has been typeset from a  $\text{\TeX}/\text{\LaTeX}$  file prepared by the author.