A Landau fluid model for dispersive magnetohydrodynamics

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ABSTRACT

A monofluid model including Landau damping, generalized Ohm's law and FLR corrections is presented for a magnetized collisionless electron-proton plasma with distribution functions close to bi-Maxwellians. Applications to the dynamics of weakly nonlinear dispersive MHD waves are discussed.

"Magnetic fields in the Universe: from laboratory and stars to the primordial structures"

Angra dos Reis, November 28, 2004

I. Introduction

The magnetosheath (buffer between the earth bow shock and the magnetopause) plays an important role: decrease the impact of solar activity on the earth environment.

- There is a wide spectrum of low frequency modes (Alfvén, slow and fast magnetosonic, mirror). Cluster spacecrafts allow one to determine k-spectra and clearly identify modes (Sahraoui et al. 2004).

- Size of perturbuations can be smaller than the ion gyroradius.

- The plasma is relatively warm and collisionless.

- Landau damping and finite Larmor radius corrections play an important role.

- There is evidence of coherent solitonic structures in the form of magnetic holes and shocklets, whose origin is still debated (Tsurutani et al. 2004).

Another context

Turbulence at small scale in the warm and hot ionized phases of the Interstellar medium, in HII regions or in the Galactic halo.

Several questions can be asked:

- What is the connection between this small-scale turbulence and the cascade originating from the large-scale motions
- What is the role of these fluctuations on cosmic ray scattering and acceleration or vice-versa the impact of cosmic ray streaming instabilities in generating such fluctuations
- What is the minimal reasonable model for understanding short-scale fluctuations?
- What modes and structures are involved? what are the 3D spectra?
- What is the dominant dissipation mechanism and the resulting heating? How does it affect chemestry?

Electron density irregularities (scintillation measurements) extends to (10-100 km), beyond the ion Larmor radius and well below the ion-neutral and Coulomb mean free paths:
ion gyro-radius $\approx 10^8/B(\mu G)$ cm.
ion-neutral mean free path: $\approx 10^{15}/n$ cm.
Coulomb mean free path: $\approx 10^{14}/n_i$ cm.

The model should include

- equations for the ions, electrons and neutrals
- coupling to dust, cosmic rays, radiation
- proper account of dispersive and kinetic effects of plasma waves
- inclusion of (few) collisions

Which tool?

• Description of intermediate-scale dynamics by usual MHD is questionable.

• Numerical integration of Vlasov-Maxwell or gyrokinetic equations often beyond the capabilities of present day computers.

• This suggests the development of a reduced description that retains most of the aspects of a FLUID MODEL but INCLUDES REALISTIC APPROXIMATIONS OF THE PRESSURE TENSOR AND WAVE-PARTICLE RESONANCES.

Should remain simple enough to allow numerical simulation of 3D dispersive MHD turbulence with realistic dissipation.

⋆ Gyrofluids: hydrodynamic moments obtained from gyrokinetic equations. Capture high order FLR but need a specific closure and are written in a local reference frame.

⋆ Landau fluids [Hammett and co-authors (1990s)]: monofluid taking into account wave-particle resonances in a way consistent with linear kinetic theory.

II. Outline of the method

• Goal: Extend Landau-fluid model, to reproduce the weakly nonlinear dynamics of dispersive MHD (magnetosonic and Alfvén) waves whatever their direction of propagation, in particular of kinetic Alfvén waves (KAW) with $k\rho_L \leq 1$, by retaining FLR corrections and a generalized Ohm's law in addition to Landau damping.

• Starting point: Vlasov-Maxwell (VM) equations.

• Small parameter: ratio between the ion Larmor radius and the typical (smallest) wavelength. Field amplitudes also supposed to be small.

• Main problem: Exact hydrodynamic equations are obtained by taking moments of VM equations. The hierarchy must however be closed and the main work resides in a proper determination of the pressure tensor.

• Assumptions: Homogeneous equilibrium state with bi-Maxwellian distribution functions.

III. The equations:

- From Vlasov-Maxwell equations, derive a hierarchy of moment equations for each particle species r.

- Monofluid approximation.

$$\partial_t \rho + \nabla \cdot (u\rho) = 0$$

$$\partial_t (\rho u) + \nabla \cdot (\rho u \otimes u) + \nabla \cdot \mathbf{p} - \frac{1}{c} j \times b = 0$$

where $u = \frac{1}{\rho}\sum_r \rho_r u_r$, $\rho = \sum_r \rho_r$ and $j = \dfrac{c}{4\pi}\nabla \times b$. On has $\mathbf{p} = \sum_r \mathbf{p}_r$ where the pressure tensor $\mathbf{p}_r$ is defined in terms of the deviation from the barycentric velocity. Up to subdominant terms,

$$\partial_t \mathbf{p}_r + \nabla \cdot (u\mathbf{p}_r + \mathbf{q}_r) + [\mathbf{p}_r \cdot \nabla u + \frac{q_r}{m_r c} b \times \mathbf{p}_r]^{\mathcal{S}} = 0.$$

Induction equation with Hall-effect and electron pressure:

$$\partial_t b - \nabla \times (u \times b) = -\frac{m_i c}{q_i} \nabla \times [\frac{1}{4\pi\rho}(\nabla \times b) \times b - \frac{1}{\rho}\nabla \cdot \mathbf{p}_e].$$

Two problems:

(a) Heat fluxes require a CLOSURE APPROXIMATION.

(b) Equation for $\mathbf{p_r}$ involves small time scale.

- Closure: Reductive perturbative expansion on the Vlasov-Maxwell equations associated with the various types of (long) MHD waves provides ASYMPTOTICALLY EXACT (possibly nonlocal) RELATIONS BETWEEN THE HEAT FLUXES AND LOWER ORDER MOMENTS, FROM WHICH WE INFER GENERAL CLOSURE ASSUMPTIONS.

Advantages:

Relative simplicity: isolates waves of different kinds and get rid of irrelevant terms.

Rigor: expansion in terms of a single small parameter.

Bonus: allows to test the equations in the weakly nonlinear regime.

Provides relations between heat fluxes and lower order moments.

Modeling the heat fluxes:

The gyrotropic and non-gyrotropic contributions to the heat fluxes $\mathbf{q}_r$ are separated by writing $\mathbf{q}_r = \mathbf{q}_r^G + \mathbf{q}_r^{NG}$ with

$$q^G_{ijk,r} = q_{\|r}\widehat{b}_i\widehat{b}_j\widehat{b}_k + q_{\perp r}(\delta_{ij}\widehat{b}_k + \delta_{ik}\widehat{b}_j + \delta_{jk}\widehat{b}_i - 3\widehat{b}_i\widehat{b}_j\widehat{b}_k),$$

Linear (or weakly nonlinear) kinetic theory IN THE LONG-WAVE ASYMPTOTICS:

$$\frac{q^{(1)}_{\|r}}{v_{th,r}p^{(0)}_{\|r}} = \frac{c_r(c_r^2 - 3 + \mathcal{W}_r^{-1})}{c_r^2 - 1 + \mathcal{W}_r^{-1}}\frac{T^{(1)}_{\|r}}{T^{(0)}_{\|r}}$$

$$\frac{q^{(1)}_{\perp r}}{v_{th,r}p^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}}\frac{c_r\mathcal{W}_r}{1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}}\mathcal{W}_r}\frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}}c_r\mathcal{W}_r|b^{(1)}|.$$

Plasma response function $\mathcal{W}_r \equiv \mathcal{W}(c_r) = \frac{1}{\sqrt{2\pi}}\mathrm{P}\int\frac{\zeta e^{-\zeta^2/2}}{\zeta - c_r}d\zeta + \sqrt{\frac{\pi}{2}}c_r e^{-c_r^2/2}\mathcal{H}$,

where $c_r = -\frac{1}{v_{th,r}}\partial_t\partial_x^{-1}$, $v_{th,r} = \sqrt{T^{(0)}_{\|r}/m_r}$ and $\mathcal{H}$ is the Hilbert transform.

Replace the plasma response function by two or four poles Padé approximants, in a way that leads to initial value problems for the heat fluxes.

Heat flux closure:

$$\left(\frac{d}{dt}+\frac{v_{th,r}}{\sqrt{\frac{8}{\pi}}(1-\frac{3\pi}{8})}\mathcal{H}\nabla_{\parallel}\right)\frac{q_{\parallel r}}{v_{th,r}p_{\parallel}^{(0)}}=\frac{1}{1-\frac{3\pi}{8}}v_{th,r}\nabla_{\parallel}\frac{T_{\parallel r}}{T_{\parallel r}^{(0)}}$$

$$\left(\frac{d}{dt}-\sqrt{\frac{\pi}{2}}v_{th,r}\mathcal{H}\nabla_{\parallel}\right)\frac{q_{\perp r}}{v_{th,r}p_{\perp r}^{(0)}}=-v_{th,r}\nabla_{\parallel}\left(\frac{T_{\perp r}}{T_{\perp r}^{(0)}}+\left(\frac{T_{\perp r}^{(0)}}{T_{\parallel r}^{(0)}}-1\right)\frac{|b|}{B_0}\right),$$

with $p_{\parallel r}=nT_{\parallel r}$ and $p_{\perp r}=nT_{\perp r}$.

To properly describe oblique and kinetic Alfvén waves, these equations have to be modified by including the effect of parallel current. Nongyrotropic heat flux components are also to be modeled by fitting with the kinetic theory.

Equations for the pressures

Applying the trace operator and the contracting with $\widehat{b}\otimes\widehat{b}$ on both sides of the pression equation gives equations for the gyrotropic pressures

$$\partial_t p_{\perp r} + \nabla\cdot(u\, p_{\perp r}) + p_{\perp r}\nabla\cdot u - p_{\perp r}\,\widehat{b}\cdot\nabla u\cdot\widehat{b} + \frac{1}{2}(\mathrm{tr}\,\nabla\cdot\mathbf{q}_r - \widehat{b}\cdot(\nabla\cdot\mathbf{q}_r)\cdot\widehat{b})$$
$$+\frac{1}{2}(s_{1r} - s_{2r} + s_{3r}) = 0$$
$$\partial_t p_{\| r} + \nabla\cdot(u\, p_{\| r}) + 2p_{\| r}\,\widehat{b}\cdot\nabla u\ \cdot\widehat{b} + \widehat{b}\cdot(\nabla\cdot\mathbf{q}_r)\cdot\widehat{b} + s_{2r} - s_{3r} = 0.$$

CGL eqs. with heat fluxes and coupling to non-gyrotropic components
$s_{1r} = \mathrm{tr}\,(\boldsymbol{\pi}_r\cdot\nabla u)^S$, $s_{2r} = (\boldsymbol{\pi}_r\cdot\nabla u)^S : \widehat{b}\otimes\widehat{b}$, $s_{3r} = \boldsymbol{\pi}_r : \frac{d}{dt}(\widehat{b}\otimes\widehat{b})$.

For weak perturbations of an equilibrium state with uniform density, gyrotropic pressures and uniform magnetic field, s_{1r}, s_{2r} and s_{3r} are SUBDOMINANT at all the relevant orders of the present analysis.

Both gyrotropic and non-gyrotropic heat flux components a priori contribute to the gyrotropic components of $\nabla\cdot\mathbf{q}_r$.

Finite Larmor radius corrections:

The non-gyrotropic part of the pressure satisfies

$$\boldsymbol{\pi}_r \times \widehat{b} - \widehat{b} \times \boldsymbol{\pi}_r = \overline{\mathbf{k}}_r$$

where $\quad \overline{\mathbf{a}} = \mathbf{a} - \frac{1}{2}\mathbf{a} : (\mathbf{I} - \widehat{b} \otimes \widehat{b})(\mathbf{I} - \widehat{b} \otimes \widehat{b}) - (\mathbf{a} : \widehat{b} \otimes \widehat{b})\widehat{b} \otimes \widehat{b},$

This equation is solved perturbatively

- $m_e/m_i \ll 1$: only non-gyrotropic corrections due to ions are relevant.
- Leading order $\boldsymbol{\pi}_p^{(1)}$ reproduces Yajima's (1966) result

$$\pi^{(1)}_{p\,xx} = -\pi^{(1)}_{p\,yy} = -\frac{p_{\perp p}}{2\Omega_p}(\partial_y u_x + \partial_x u_y)$$

$$\pi^{(1)}_{p\,zz} = 0$$

$$\pi^{(1)}_{p\,xy} = \pi^{(1)}_{p\,yx} = -\frac{p_{\perp p}}{2\Omega_p}(\partial_y u_y - \partial_x u_x)$$

$$\pi^{(1)}_{p\,yz} = \pi^{(1)}_{p\,zy} = \frac{1}{\Omega_p}[2p_{\|p}\partial_z u_x + p_{\perp p}(\partial_x u_z - \partial_z u_x)]$$

$$\pi^{(1)}_{p\,xz} = \pi^{(1)}_{p\,zx} = -\frac{1}{\Omega_p}[2p_{\|p}\partial_z u_y + p_{\perp p}(\partial_y u_z - \partial_z u_y)].$$

IV. Validation

- For parallel Alfvén waves, the long-wave reductive perturbative expansion performed on the Landau-fluid model reproduces KDNLS equations derived from Vlasov-Maxwell, up to the replacement of the plasma response function $\mathcal{W}$ by the corresponding two- or four-pole approximants.

- For Alfvén waves at finite angle of propagation, FLR corrections of order $1/\Omega_p^2$ are to be retained. The governing equation is linear and reads, assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$, (adiabatic protons and isothermal electrons) and $\beta \ll 1$ (ξ: stretched coordinate along the propagation)

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}[\frac{\cos^3\alpha}{\sin^2\alpha} + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\cos^3\alpha(\tan^2\alpha + \frac{1}{\tan^2\alpha})\mathcal{H}]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0,$$

- For Kinetic Alfvén waves ($\cos^2\alpha \ll \beta$), agree with Akhiezer et al. '75, Hasegawa and Chen '76).

$$\partial_\tau \frac{b_y}{B_0} + \frac{v_A^3}{2\Omega_p^2}\cos\alpha[-\beta(1 + \frac{3}{4}\frac{T_p^{(0)}}{T_e^{(0)}}) + \sqrt{\beta}\sqrt{\frac{\pi}{2}}\sqrt{\frac{m_e}{m_p}}\mathcal{H}]\partial_{\xi\xi\xi}\frac{b_y}{B_0} = 0.$$

- For magnetosonic waves, Landau damping rate is (assuming $\frac{m_e}{m_p} \ll \beta \ll \frac{T_e}{T_p}$)

$$\gamma = -\sqrt{\beta}\sqrt{\frac{\pi}{8}}\sqrt{\frac{m_e}{m_p}}\frac{\sin^2\alpha}{\cos\alpha}\frac{(\omega^2 - \beta\cos^2\alpha)^2 + \beta^2\cos^4\alpha}{(2\omega^2 - \beta - 1)(\omega^2 - \beta\cos^2\alpha)},$$

an expression identical to that found by a direct derivation from the Vlasov-Maxwell equations. The long-wave equation is KdV+damping term.

VI. Numerical simulations of the model in a slab geometry

• Landau dissipation for an oblique fast wave.

To approach the asymptotic regime we take $T_e/T_p = 10$, and prescribe small values of β. An agreement of the order of 1 % percent is obtained in the regime where β is such that Landau dissipation dominates the dynamics.

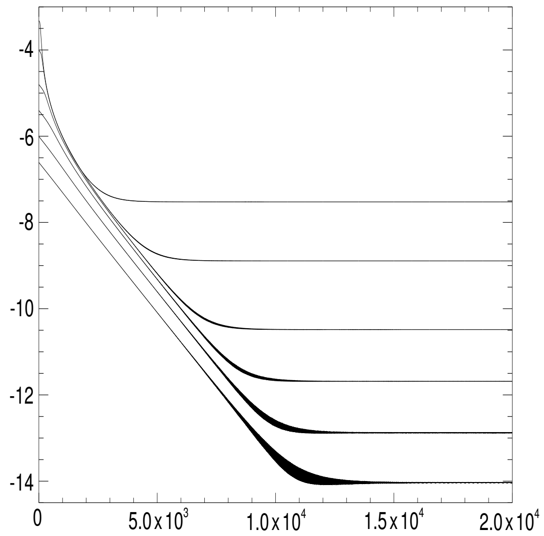

Figure 1: Time evolution (in lin-log scales) of magnetosonic wave energy $\frac{1}{2}\int[\rho(u_x^2+\widetilde{u}_z^2)+(b_z-B_0\sin\alpha)^2]dx$, assuming $R_p = 7.\ 10^{-3}$.

An Alfvén wave is generated that rapidly evolves to a large-scale profile, the small-scale oscillations being damped at a rate that scales like k^3/R_p^2. After a while, the magnetosonic contribution to the solution has been almost completely dissipated and the resulting state can be viewed as an Alfvén wave essentially insensitive to Landau damping.

This evolution results in an increase of $T_{\|e}$ by about 20% in the case of a wave of initial amplitude $a_0 = 2.2\ 10^{-2}$. The associated time scale corresponds to the formation and dissipation of the high frequency modes in the dispersive shocks.

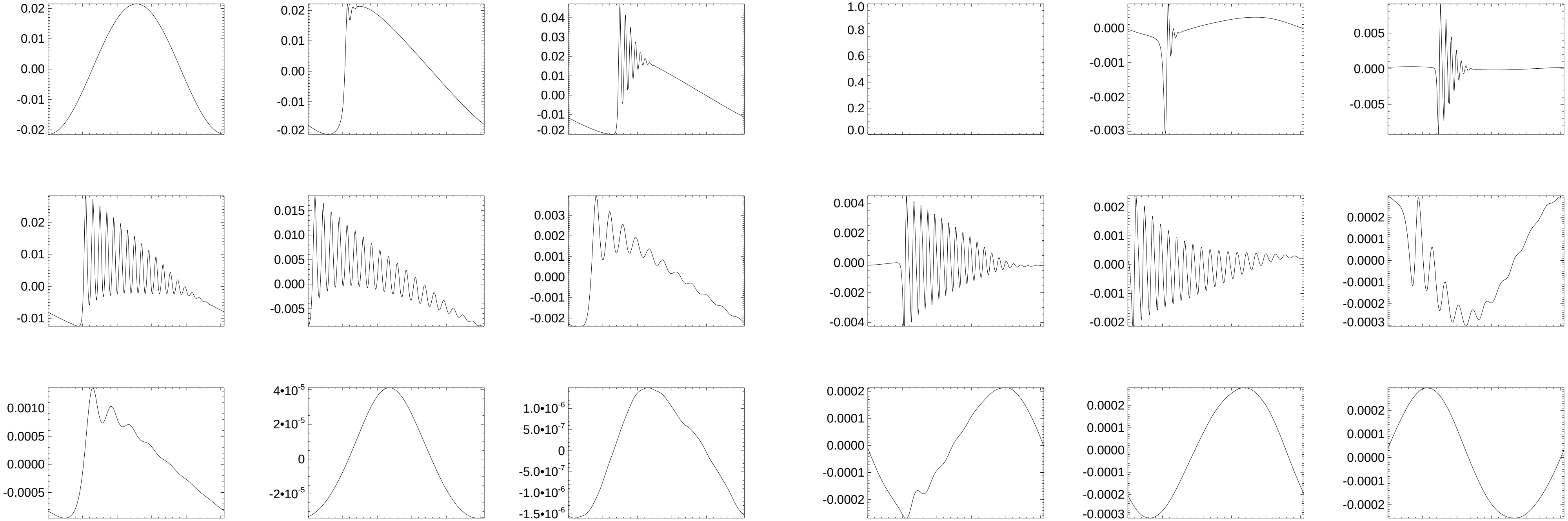

Figure 2: Snapshots at times $t = 1$, 32, 45, 101, 150, 501, 1002, 4000 and 17000 of the velocity component $\widetilde{u}_z$ (left) and u_y (right) for a magnetosonic wave of initial amplitude $a_0 = 2.2\ 10^{-2}$, with $\beta = 10^{-2}$ and $R_p = 7\ 10^{-3}$.

- Small dissipation limit

When dissipation is small enough to permit a balance between dispersion and nonlinearity, a reductive perturbative expansion leads to a perturbed Korteweg de Vries equation (Janiki $et\ al.$)

To simulate such a regime, it is appropriate to decrease the scale separation (taking $R_p = 10^{-1}$) and increase the wave amplitude (with $a_0 = 10^{-2}$).

We first observe, on the typical steepening time scale, the formation of solitonic structures with a hump for the velocity u_z and density depressions correlated with magnetic holes.

Eventually, on a time 15 times larger, the solution profile becomes significantly distorted and progressively evolves to a quasi-stationary wave. In this regime, the dissipation of the magnetosonic wave remains very weak during the whole simulation and the Alfvén waves remain subdominant.

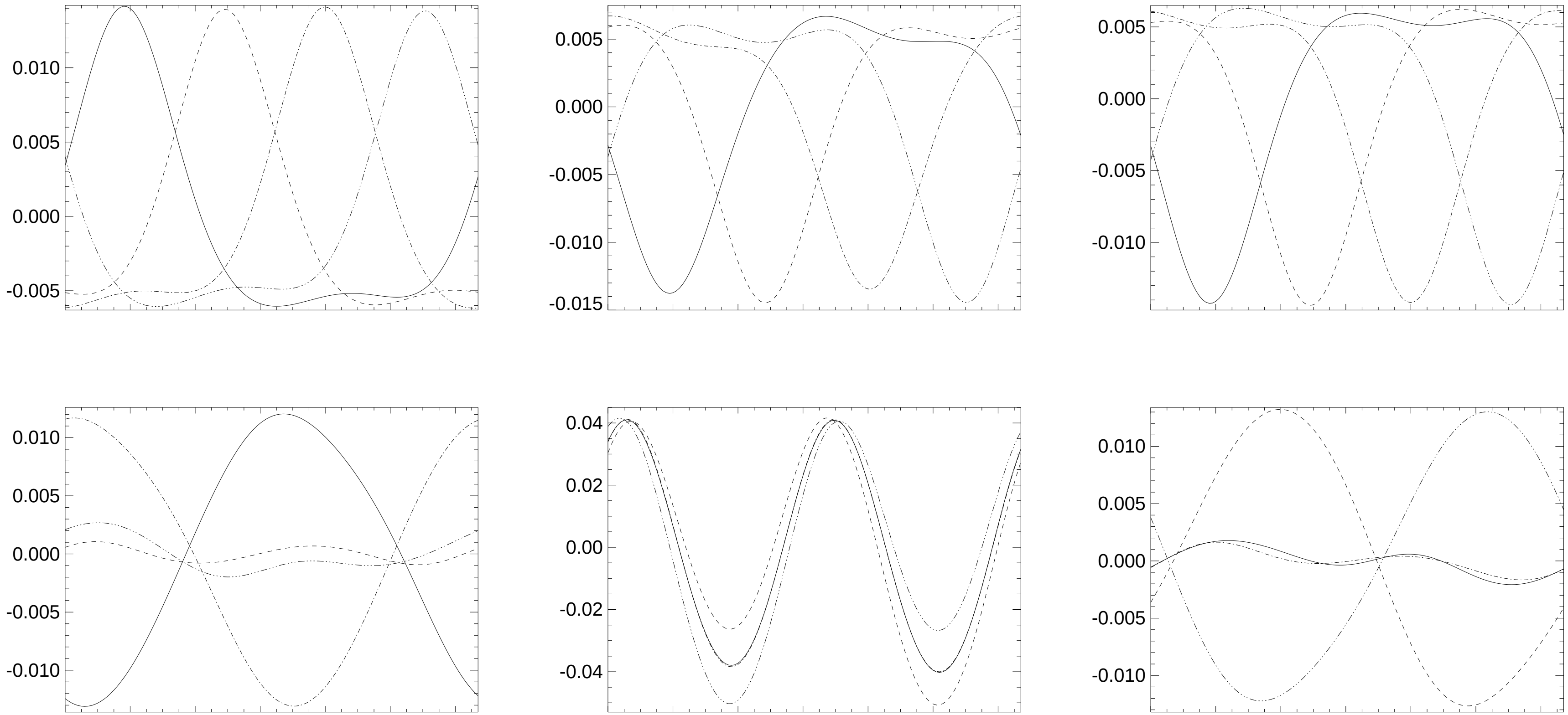

Figure 3: Profiles of the $\widetilde{u}_z$ component (left), of the density perturbation (middle) and of the magnetic field perturbation (right) at successive times (denoted by solid, dashed, dotted-dashed and triple-dotted-dashed lines) separated by 1.5 units starting at $t = 104.3$ (top) and at $t = 1502.3$ (bottom), in the case $a_0 = 10^{-2}$, $R_p = 10^{-1}$ and $\beta = 10^{-3}$.

• Parametric instabilities of parallel propagating Alfvén waves

Non dispersive regime

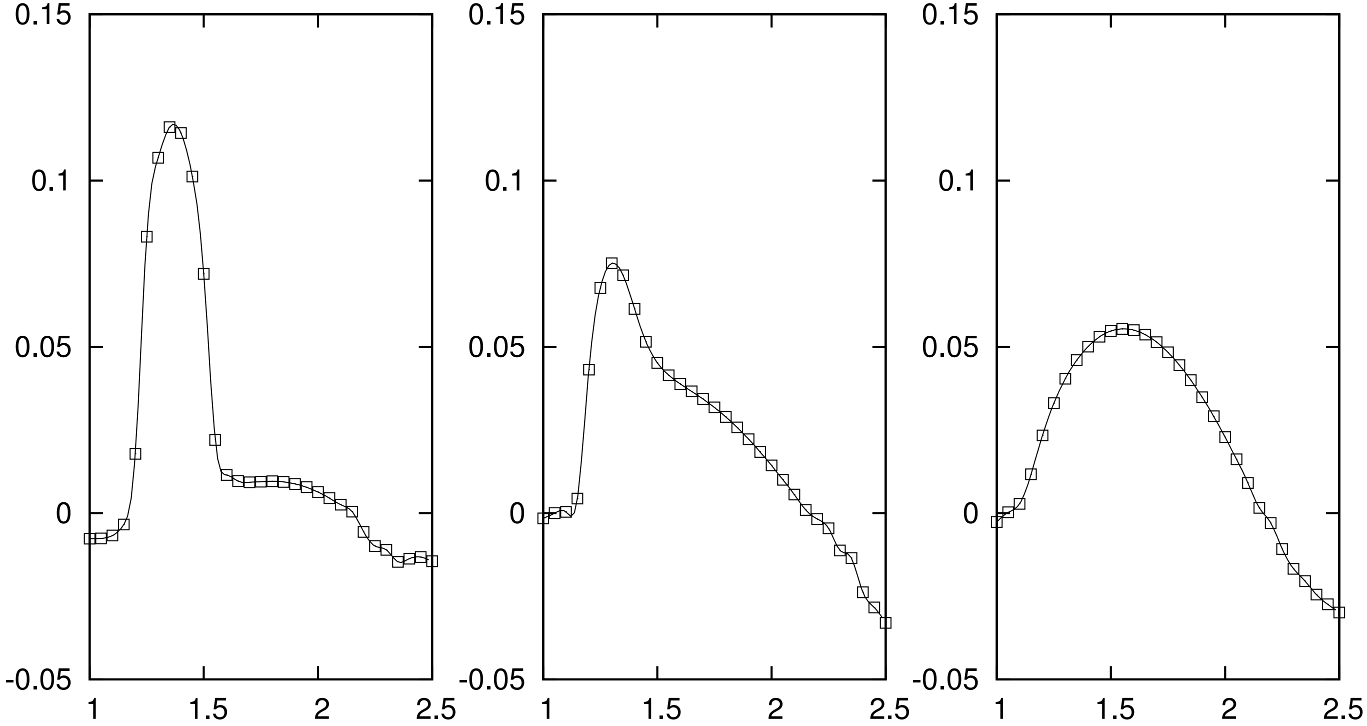

Figure 4: Growth rates for $\beta = 0.6$ and $T_e/T_i = 33$ (left), $T_e/T_i = 5$ (middle) and $T_e/T_i = 1$ (right).

Reduction of the instability growth rate and broadening of its spectral range, as T_i is increased. Compares very well with drift-kinetic treatment of Inhester (1990).

Saturation by Landau damping, with very small excitation of higher harmonics. Ions are predominantly heated in the nonlinear phase.

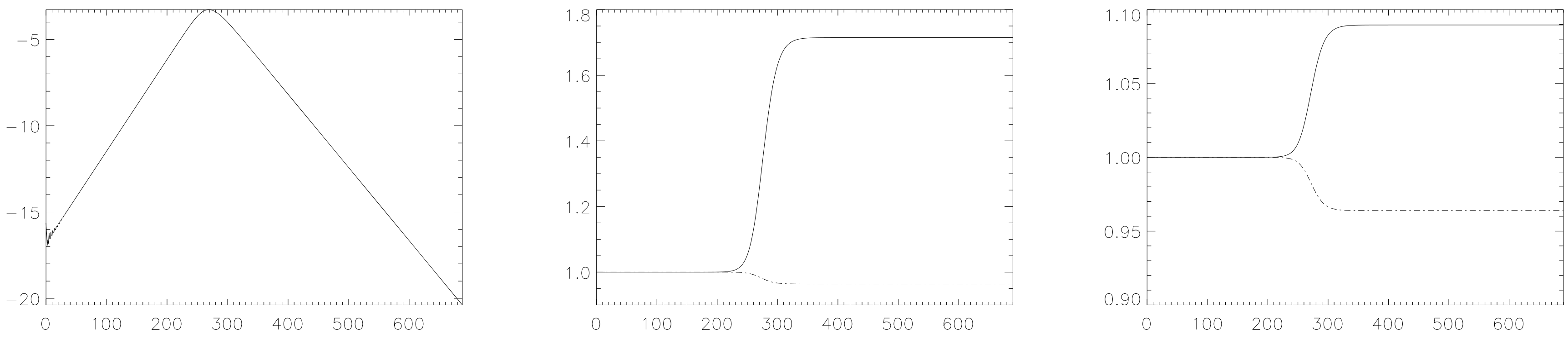

Figure 5: Time evolution of the amplitude logarithm for density mode k=1.5 in run with $T_e = T_i$ (left), time evolution of ion parallel (solid) and ion perpendicular (dashed-dotted) temperature (middle), same for electrons (right).

Dispersive regime

For the right-handed mode at $\beta = .4$ and a pump amplitude $B_0 = 0.1$, an algebraic inverse cascade develops. EXCITATION IS TRANSFERED TO LARGER AND LARGER SCALES WHILE THE DIRECTION OF PROPAGATION OF THE WAVE SWITCHES ALTERNATIVELY AT EACH STEP OF THE PROCESS.

At $\beta = .45$, a pump amplitude $B_0 = 0.5$ and $T_e = 0$ the cascade extends to $k = 1$. Each step is associated with a parallel ion temperature increase.
Electrons remain cold, which supports the isothermal fluid description of electrons in the hybrid code (Vasquez 1995).

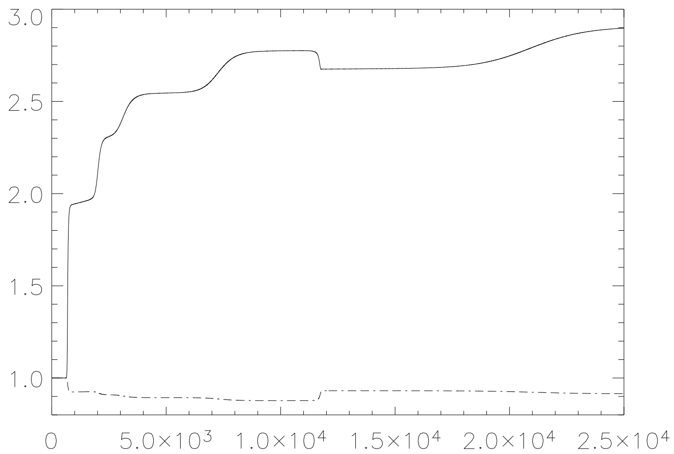

Figure 6: Ion temperature evolution for a run with a right-handed wave with amplitude $b_0 = .5$, in a plasma with $\beta = .45$ and $T_e = 0$.

COMPARISON WITH FLUID THEORY

Decay instability can persist at high values of β. Taking $R_p = 0.1$, $b_0 = .5$ and $\beta = 5$, a decay instability is clearly visible at early time, that leads to the dominance of the (backward) $m = 4$ mode, whereas the fluid theory predicts a modulational instability.

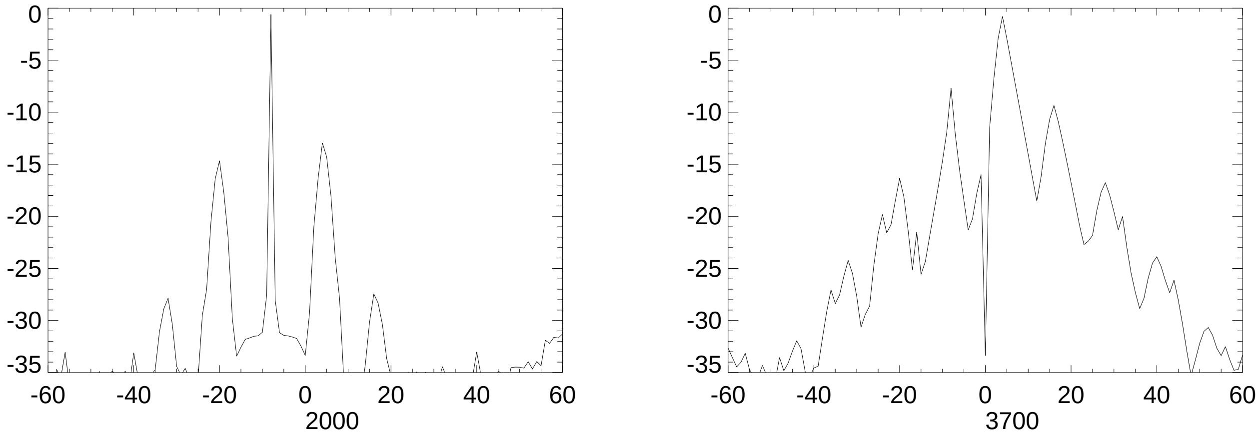

Figure 7: Spectral density for the complex quantity $b_+ = b_x + ib_y$ in the linear phase of the decay instability at $t = 2000$ (left) and in the nonlinear phase at $t = 3700$ (right) for the run with a right-hand polarized wave with amplitude $b_0 = .5$, in a plasma with $\beta = 5$, $R_p = .1$ and $T_i/T_e = 1.5$.

When taking $R_p = 1$ a small-scale instability associated with FLR terms is observed (limit of validity of the model: unsufficient scale separation).

For the left-handed mode at $\beta = 1.5$ kinetic effects induce a modulational instability (Mjølhus & Wyller 1988) whereas fluid theory predicts only beat instability.
This is verified numerically, choosing $k_0 = 0.408$ (length unit being the ion inertial length) and a pump amplitude $B_0 = 0.3$. Ions significantly heated but not electrons.

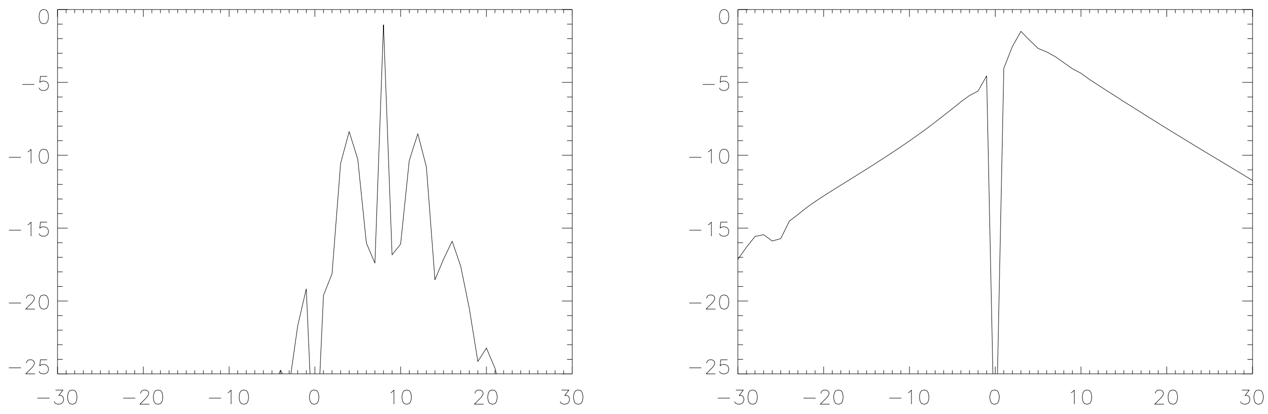

Figure 8: Spectral density for the complex quantity b_+ in the linear phase of the modulational instability at $t = 2000$ (left) and in the nonlinear phase at $t = 3700$ (right) for the run with LH wave with $B_0 = 0.3$, $\beta = 1.5$ and $T_e = 2T_p$.

When $T_\perp > T_\parallel$ mirror modes may become unstable (Hasegawa).
The description of these modes requires an accurate modeling of the kinetic effects .

When $T_e = T_i$, the condition for the instability reads $\frac{p_\perp^{(0)}}{p_\parallel^{(0)}} > 1 + \frac{1}{p_\perp^{(0)}} \frac{B_0^2}{8\pi}$. Assuming an angle of propagation such that $\cos\alpha = 0.01$ and $\beta = 1$ with $R_p = 10^{-10}$ we numerically calculate the growth rate. The agreement is excellent.

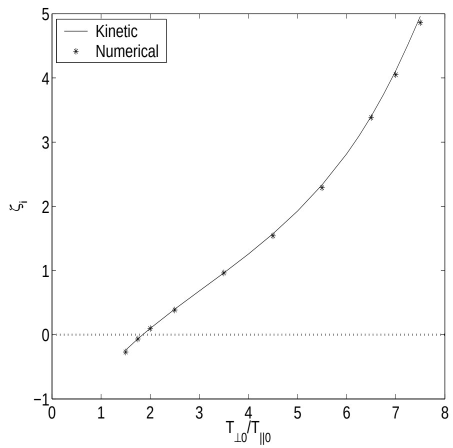

Figure 9: Mirror mode growth rate predicted by the kinetic theory and given by time integration of the model equations, versus the equilibrium temperature anisotropy for a plasma with $\beta = 1$ and equal temperatures for electrons and protons.

Perspectives

• Benchmark the model by comparison with PIC and possibly Vlasov-Maxwell simulations.

• Explore the nonlinear stage of parametric instabilities.

• Modelisation of coherent structures (magnetic holes and shocklets) observed in the solar wind and magnetosheath.

• Simulation of dispersive Alfvén wave turbulence:

⋆ Generation of KAW at small scales: importance of higher-order FLR corrections (that are to be described in a computationally manageable way). Also important to reproduce the mirror instability at small scale.

⋆ Self-consistent computation of turbulent dissipation.
⋆ Determination of the fast wave spectrum (important for cosmic ray scattering).
⋆ Possible emergence of coherent structures.

• Treat electrons as a Landau fluid in hybrid simulations.

• Explore the possible description of nonlinear Landau damping.

References:

T. Passot & P.L. Sulem:

• A long-wave model for Alfvén wave trains in a collisionless plasma:

I. Kinetic theory, *Phys. Plasmas* **10**, 3887 (2003).

II. A Landau-fluid approach, *Phys. Plasmas* **10**, 3906 (2003).

• Filamentation instability of long Alfvén waves in warm collisionless plasmas, *Phys. Plasmas* **10**, 3914 (2003).

• A fluid description for Landau damping of dispersive MHD waves, *Nonlinear Proc. Geophys.* **11**, 245 (2004).

• A Landau fluid model for dispersive magnetohydrodynamics, *Phys. Plasmas*, **11**, 5173-5189 (2004).

• G. Bugnon, R. Goswami, T. Passot & P.L. Sulem:

Towards fluid simulations of dispersive MHD waves in a warm collisionless plasma, in "Dynamical Processes in Critical Regions of the Heliosphere", R. von Steiger and M. Gedalin eds., *Advances in Space Research*, Elsevier (2004), in press.

• G. Bugnon, T. Passot & P.L. Sulem:

Landau-fluid simulations of Alfvén-wave instabilities in a warm collisionless plasma, *Nonlinear Proc. Geophys.*, to appear.

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