Long-Alfvén-wave trains in collisionless plasmas. II. A Landau-fluid approach

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A Hall Landau-fluid model where the fluid hierarchy is closed by nonlocal dynamical equations for the heat fluxes is constructed, based on a weakly nonlinear description of long parallel Alfvén waves in a collisionless plasma. This model which is shown to accurately reproduce the above asymptotic regime, can be viewed as a first step towards the simulation of dispersive Alfvén wave turbulence with a realistic dissipation.

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I. INTRODUCTION

In the companion paper[1] (hereafter referred to as Paper I), a reductive perturbative expansion of the Vlasov-Maxwell equations led to a closed system of equations for the dynamics of long Alfvén wave trains propagating along a strong ambient magnetic field in a collisionless plasma. This analysis revisited an early work by Rogister[2] that was restricted to localized pulses, and provided an extension to Alfvén wave trains. An interesting point is that Rogister's long-wave model, now referred to as the kinetic derivative nonlinear Schrödinger equation (KDNLS)[3] was reproduced by Mjølhus and Wyller[4] who performed a long-wave expansion on the Hall-MHD equations including finite Larmor radius corrections[5] and supplemented by a perturbative kinetic computation of the gyrotropic pressure components based on the guiding center approximation. This reductive perturbative expansion that can be viewed as providing an exact closure of the fluid hierarchy including Landau damping is however specific to the dynamics of unidirectional long Alfvén waves.

Semi-heuristic closures were proposed at the level of the heat flux or higher order moments, incorporating linear Landau damping (but not particle trapping).[6] These Landau-fluid models were extended to magnetohydrodynamics by Snyder, Hammett and Dorland[7] (hereafter referred to as SHD) where the analysis is based on the guiding center approximation. The aim of the present paper is to bridge the gap between such models and the asymptotically exact closures provided by a long-wave reductive perturbative expansion. Section II describes the (unclosed) hierarchy of fluid equations that governs the plasma dynamics when finite Larmor radius corrections and the Hall effect are retained. Section III discusses the multidimensional kinetic derivative nonlinear Schr̈odinger equations derived in Paper I by means of a reductive perturbative expansion of the Vlasov-Maxwell equations, and provides additional relations between the moments that will be useful in the construction of Landau-fluid closures. Section IV revisits the closure analysis of the fluid hierarchy presented by SHD. By extrapolating relations that are exact in the long-wave asymptotics by means of algebraic fits of the plasma response function, it provides a simpler description of the heat flux dynamics, while keeping the same accuracy. Section V demonstrates that the resulting Hall-Landau fluid model provides a very accurate description of the nonlinear dynamics of long dispersive Alfvén waves. We conclude that this model should provide an efficient tool for realistic simulations of dispersive Alfvén wave turbulence.

II. FLUID EQUATIONS

Starting with the Vlasov-Maxwell equations and deriving the equations satisfied by the successive moments of the distribution function (see paper I for a precise definition of the notations), one classically derives for each species an exact hierarchy of fluid equations for the density $\rho_r = m_r n_r \int f_r d^3v$, the hydrodynamic velocity $u_r = \int v f_r d^3v / \int f_r d^3v$, the pressure tensor $P_r = m_r n_r \int (v-u_r)\otimes(v-u_r) f_r d^3v$ and the heat flux tensor $Q_r = m_r n_r \int (v-u_r)\otimes(v-u_r)\otimes(v-u_r) f_r d^3v$, in the form

$$\partial_t \rho_r + \nabla\cdot(u_r\rho_r) = 0 \quad (1)$$

$$\partial_t u_r + u_r\cdot\nabla u_r + \frac{1}{\rho_r}\nabla\cdot P_r - \frac{q_r}{m_r}(e + \frac{1}{c}u_r\times b) = 0 \quad (2)$$

$$\partial_t P_r + \nabla\cdot(u_r P_r + Q_r) + 2\Big[P_r\cdot\nabla u_r + \frac{q_r}{m_r c} b\times P_r\Big]^{\mathcal{S}} = 0 \quad (3)$$

where $[\cdot]^{\mathcal{S}}$ holds for the symmetric part of the corresponding matrix.

When concentrating on large spatio-temporal scales compared to the gyration parameters of the particles[9], one can consider the limit $\Omega_r = \dfrac{q_r B_0}{m_r c} \to \infty$ (where B_0 is the amplitude of the ambient field). This procedure is equivalent to first taking this limit at the level of the Vlasov equation to derive the equation governing the guiding center distribution function and then constructing the corresponding moment hierarchy.[8] In this limit $b \times P_r^{\mathcal{S}}$ must vanish to leading order. Expanding the pressure tensor in the form $P_r = P_r^0 + P_r^{(1)} + \cdots$, the above condition implies that $P_r^{(0)}$ has the gyrotropic form

$$P_r^{(0)} = p_{\perp r}(I - \widehat{b} \otimes \widehat{b}) + p_{\parallel r}\widehat{b} \otimes \widehat{b} \tag{4}$$

where I is the identity matrix, $\widehat{b}$ the unit vector along the local magnetic field b. The coefficients $p_{\perp r}$ and $p_{\parallel r}$ are the transverse and parallel gyrotropic pressures associated to species r.

At the next order

$$\nabla \cdot Q_r^{(0)} + LP_r^{(0)} = P_r^{(1)} \times b - b \times P_r^{(1)} \tag{5}$$

where L denotes a linear operator. The solvability of this equation for $P_r^{(1)}$ requires that the left-hand-side obeys the two conditions: (i) its trace vanishes; (ii) its contraction with $\widehat{b}$ on the right and left sides also vanishes. Denoting by $q_{\parallel r}$ and $q_{\perp r}$ the parallel and perpendicular components (relatively to the local magnetic field) of the gyrotropic heat flux tensor

$$Q_{ijk,r}^{(0)} = q_{\parallel r}\widehat{b}_i\widehat{b}_j\widehat{b}_k + q_{\perp r}(\delta_{ij}\widehat{b}_k + \delta_{ik}\widehat{b}_j + \delta_{jk}\widehat{b}_i - 3\widehat{b}_i\widehat{b}_j\widehat{b}_k), \tag{6}$$

one gets[7]

$$\partial_t p_{\parallel r} + \nabla \cdot (u_r\, p_{\parallel r}) + \nabla \cdot (\widehat{b}\, q_{\parallel r}) + 2p_{\parallel r}\, \widehat{b} \cdot \nabla u_r\, \cdot \widehat{b} - 2q_{\perp r}\nabla \cdot \widehat{b} = 0, \tag{7}$$

$$\partial_t p_{\perp r} + \nabla \cdot (u_r\, p_{\perp r}) + \nabla \cdot (\widehat{b}\, q_{\perp r}) + p_{\perp r}\nabla \cdot u_r - p_{\perp r}\, \widehat{b} \cdot \nabla u_r \cdot \widehat{b} + q_{\perp r}\nabla \cdot \widehat{b} = 0. \tag{8}$$

At this step, the fluid system is unclosed, the heat flux components obeying dynamical equations that involve fourth-order moments.[7]

III. DYNAMICS OF LONG-ALFVÉN-WAVE TRAINS

A. The reductive perturbation expansion

As already mentioned, the closure problem faced when deriving the moment hierarchy disappears when the analysis is restricted to the dynamics of long Alfvén waves. Indeed, the long-wave reductive perturbative expansion performed on Paper I led to a closed system of equations for the leading order dynamics of the transverse magnetic field $\epsilon b_\perp^{(0)}$ and the fluctuating longitudinal magnetic disturbance $\epsilon^2 b_\parallel^{(1)}$, in terms of the stretched longitudinal variable $\xi = \epsilon^2(x - \lambda t)$, of the transverse ones $\eta = \epsilon^3 y$ and $\zeta = \epsilon^3 z$, and of the slow time $\tau = \epsilon^4 t$, in the form

$$\Big(\partial_\tau + \langle U\rangle\partial_\xi\Big)b_\perp^{(0)} + \partial_\xi\Big(\frac{\widetilde{P}b_\perp^{(0)}}{2\lambda\rho^{(0)}}\Big) - \frac{B_0}{2\lambda\rho^{(0)}}\nabla_\perp\widetilde{P} + \delta\,\partial_{\xi\xi}(\widehat{x} \times b_\perp^{(0)}) = 0 \tag{9}$$

$$\rho^{(0)}\partial_\tau\langle U\rangle = c_1\Big(\nabla_\perp \cdot \Big\langle\widetilde{P}\frac{b_\perp^{(0)}}{B_0}\Big\rangle - \langle\widetilde{A}\mathcal{K}\partial_\xi\widetilde{A}\rangle\Big) - c_2\,\langle\langle\widetilde{A}\mathcal{K}\partial_\xi\widetilde{A}\rangle\rangle, \tag{10}$$

$$\partial_\xi\widetilde{b}_\parallel^{(1)} + \nabla_\perp \cdot b_\perp^{(0)} = 0. \tag{11}$$

Here $\widetilde{P} = \Big(\frac{B_0^2}{4\pi} + 2p_\perp^{(0)} + \mathcal{K}\Big)\widetilde{A}$ refers to the leading order perturbation of the perpendicular total pressure (magnetic and kinetic). It involves the fluctuating magnetic field amplitude perturbation $\widetilde{A} = \frac{\widetilde{b}_\parallel^{(1)}}{B_0} + \frac{\widetilde{|b_\perp^{(0)}|^2}}{2B_0^2}$ and the operator $\mathcal{K} = \mathcal{N} - \mathcal{M}^2\mathcal{L}^{-1}$, where $\mathcal{L} = \sum_r \mathcal{L}_r$, $\mathcal{M} = \sum_r \mathcal{M}_r$ and $\mathcal{N} = \sum_r \mathcal{N}_r$ with

$$\mathcal{L}_r = 2\pi\frac{q_r^2 n_r}{m_r}\int_0^\infty d(\frac{v_\perp^2}{2})\,\mathcal{G}_r\ ,\ \mathcal{M}_r = 2\pi q_r n_r\int_0^\infty d(\frac{v_\perp^2}{2})\frac{v_\perp^2}{2}\,\mathcal{G}_r\ ,\ \mathcal{N}_r = 2\pi m_r n_r\int_0^\infty d(\frac{v_\perp^2}{2})\frac{v_\perp^4}{4}\mathcal{G}_r. \tag{12}$$

Here

$$\mathcal{G}_r = \mathrm{P}\int \frac{1}{v_\parallel - \lambda}\frac{\partial F_r^{(0)}}{\partial v_\parallel}\, dv_\parallel + \pi \frac{\partial F_r^{(0)}}{\partial v_\parallel}\Big|_{v_\parallel=\lambda}\mathcal{H}_\xi, \tag{13}$$

where $\mathcal{H}_\xi\{S\}$ denotes the Hilbert transform with respect to ξ. The Alfvén wave velocity λ is defined by $\lambda^2\rho^{(0)} = (\frac{|B_0|^2}{4\pi} + p_\perp^{(0)} - p_\parallel^{(0)})$ where the density $\rho^{(0)}$ and the pressure components $p_\perp^{(0)}$ and $p_\parallel^{(0)}$ at equilibrium are constructed from the velocity distribution function $F^{(0)}(v_\perp, v_\parallel)$ assumed rotationally symmetric around the direction of the ambient field $B_0\widehat{x}$ and symmetric relatively to forward and backward velocities along this direction, thus excluding the presence of equilibrium drifts.[10,11] Here simple brackets $\langle\cdot\rangle$ define averaging over the ξ variable, while double brackets $\langle\langle\cdot\rangle\rangle$ refer to the averaging in the full spatial domain. Furthermore, the constants

$$c_1 = \frac{1}{2+\beta_\perp - \beta_\parallel}\Big(\frac{12 + 18\beta_\perp + 5\beta_\perp^2}{8(1+\beta_\perp)}\Big) \quad , \quad c_2 = \frac{1}{2+\beta_\perp - \beta_\parallel}\Big(\frac{(2+\beta_\perp)^2}{8(1+\beta_\perp)}\Big) \tag{14}$$

involve the ratios $\beta_\parallel = 8\pi p_\parallel^{(0)}/B_0^2$ and $\beta_\perp = 8\pi p_\perp^{(0)}/B_0^2$ of parallel or transverse to magnetic pressure at equilibrium respectively.

The mean field $\langle U\rangle$ results from the combination

$$\langle U\rangle = \langle u_\parallel^{(1)}\rangle + \frac{\lambda}{2B_0}\langle b_\parallel^{(1)}\rangle + \frac{1}{\lambda\rho^{(0)}}(p_\parallel^{(0)} - p_\perp^{(0)})\langle A\rangle + \frac{1}{2\lambda\rho^{(0)}}(\langle p_\perp^{(1)}\rangle - \langle p_\parallel^{(1)}\rangle) \tag{15}$$

where the various terms are prescribed by the equations

$$\partial_\tau\langle u_\parallel^{(1)}\rangle = \frac{1}{\rho^{(0)}B_0}\nabla_\perp\cdot\langle\widetilde{P}b_\perp^{(0)}\rangle \tag{16}$$

$$\langle p_\perp^{(1)}\rangle + \frac{B_0^2}{4\pi}\langle A\rangle = \Gamma(\tau) \tag{17}$$

$$\frac{d}{d\tau}\Gamma(\tau) = -\frac{1}{2\lambda\rho^{(0)}}\Big(\frac{B_0^2}{4\pi} + p_\perp^{(0)}\Big)\langle\langle\widetilde{A}\mathcal{K}\partial_\xi\widetilde{A}\rangle\rangle \tag{18}$$

$$\partial_\tau\Big(\frac{\langle p_\perp^{(1)}\rangle}{p_\perp^{(0)}} - \frac{\langle\rho^{(1)}\rangle}{\rho^{(0)}} - \langle A\rangle\Big) = 0 \tag{19}$$

$$\partial_\tau\Big(\frac{\langle p_\parallel^{(1)}\rangle_\xi}{p_\parallel^{(0)}} - 3\frac{\langle\rho^{(1)}\rangle_\xi}{\rho^{(0)}} + 2\langle A\rangle\Big) = \frac{2\lambda}{p_\parallel^{(0)}}\langle\widetilde{A}\mathcal{K}\partial_\xi\widetilde{A}\rangle. \tag{20}$$

One also has the fluctuating quantities

$$\widetilde{p}_\perp^{(1)} = (2p_\perp^{(0)} + \mathcal{K})\widetilde{A} \tag{21}$$

$$\widetilde{p}_\parallel^{(1)} = (p_\parallel^{(0)} - p_\perp^{(0)})\widetilde{A} + \lambda^2(\widetilde{\rho}^{(1)} - \rho^{(0)}\widetilde{A}) \tag{22}$$

$$\widetilde{\rho}^{(1)} = (\rho^{(0)} + \mathcal{O} - \mathcal{P}\mathcal{L}^{-1}\mathcal{M})\widetilde{A}. \tag{23}$$

where $\mathcal{O} = \sum_r \mathcal{O}_r$ and $\mathcal{P} = \sum_r \mathcal{P}_r$ with

$$\mathcal{O}_r = 2\pi m_r n_r \int_0^\infty \frac{v_\perp^2}{2}\mathcal{G}_r d(\frac{v_\perp^2}{2}) \ , \ \mathcal{P}_r = 2\pi q_r n_r \int_0^\infty \mathcal{G}_r d(\frac{v_\perp^2}{2}). \tag{24}$$

Furthermore, the $(O(\epsilon^2))$ leading order fluctuating contribution to the perpendicular and parallel heat flux components are given by

$$\widetilde{q}_\perp^{(1)} = \lambda p_\perp^{(0)}\left(\frac{\widetilde{p}_\perp^{(1)}}{p_\perp^{(0)}} - \frac{\widetilde{\rho}^{(1)}}{\rho^{(0)}} - \widetilde{A}\right) \tag{25}$$

$$\widetilde{q}_\parallel^{(1)} = \lambda p_\parallel^{(0)}\left(\frac{\widetilde{p}_\parallel^{(1)}}{p_\parallel^{(0)}} - 3\frac{\widetilde{\rho}^{(1)}}{\rho^{(0)}} + 2\widetilde{A}\right). \tag{26}$$

B. Further expressions of the moments

The density, pressures and heat fluxes are given in Section III.A in terms of the magnetic perturbations only, as a consequence of the use of the quasi-neutrality condition that leads to express the electric field along the local magnetic field in terms of the magnetic perturbation [see eq. (49) of Paper I]. Such a relation is exact in the long-Alfvén wave asymptotics but appears too restrictive in more general regimes involving for example the propagation of ion-acoustic waves. One thus comes back to the distribution function given by eqs. (39)-(41) of Paper I and computes for each species r the fluctuating parts of the moments in the form (the tildes are hereafter suppressed in order to simplify the writing)

$$\rho_r^{(1)} = \rho_r^{(0)} A + \mathcal{O}_r A + \mathcal{P}_r \varphi \tag{27}$$

$$p_{\perp r}^{(1)} = 2p_{\perp r}^{(0)} A + \mathcal{N}_r A + \mathcal{M}_r \varphi \tag{28}$$

$$p_{\| r}^{(1)} = (p_{\| r}^{(0)} - p_{\perp r}^{(0)}) A + \lambda^2 \mathcal{O}_r A + \lambda^2 \mathcal{P}_r \varphi - q_r n_r \varphi \tag{29}$$

$$q_{\perp r}^{(1)} = \lambda \Big(p_{\perp r}^{(0)} A - \frac{p_{\perp r}^{(0)}}{\rho^{(0)}} \rho^{(1)} + \mathcal{N}_r A + \mathcal{M}_r \varphi \Big) \tag{30}$$

$$q_{\| r}^{(1)} = \lambda \Big(3p_{\| r}^{(0)} (A - \frac{\rho^{(1)}}{\rho^{(0)}}) - p_{\perp r}^{(0)} A + \lambda^2 (\rho_r^{(1)} - \rho_r^{(0)} A) - q_r n_r \varphi \Big). \tag{31}$$

At this step, it is convenient to eliminate the electric potential φ by algebraic combinations rather than by using the quasi-neutrality condition. One gets

$$p_{\perp r}^{(1)} = (2p_{\perp r}^{(0)} + \mathcal{N}_r) A + \mathcal{M}_r \mathcal{P}_r^{-1} (\rho_r^{(1)} - \rho_r^{(0)} A - \mathcal{O}_r A) \tag{32}$$

$$p_{\| r}^{(1)} = (p_{\| r}^{(0)} - p_{\perp r}^{(0)} + \lambda^2 \mathcal{O}_r) A + (\lambda^2 - q_r n_r \mathcal{P}_r^{-1})(\rho_r^{(1)} - \rho_r^{(0)} A - \mathcal{O}_r A) \tag{33}$$

$$q_{\perp r}^{(1)} = \lambda \Big[\mathcal{M}_r \mathcal{P}_r^{-1} \rho_r^{(1)} - p_{\perp r^{(0)}} \frac{\rho^{(1)}}{\rho^{(0)}} + \Big(p_{\perp r}^{(0)} + \mathcal{N}_r - \rho_r^{(0)} \mathcal{M}_r \mathcal{P}_r^{-1} - \mathcal{M}_r \mathcal{P}_r^{-1} \mathcal{O}_r \Big) A \Big] \tag{34}$$

$$\begin{aligned} q_{\| r}^{(1)} = \lambda \Big[&\Big(\lambda^2 - q_r n_r \mathcal{P}_r^{-1} \Big) \rho_r^{(1)} - 3p_{\| r}^{(0)} \frac{\rho^{(1)}}{\rho^{(0)}} \\ &+ \Big(3p_{\| r}^{(0)} - p_{\perp r}^{(0)} + \lambda^2 \mathcal{O}_r - (\lambda^2 - q_r n_r \mathcal{P}_r^{-1})(\rho_r^{(0)} + \mathcal{O}_r) \Big) A \Big] \end{aligned} \tag{35}$$

C. The case of a bi-maxwellian equilibrium distribution

It is possible to simplify the above general expressions for the moments by assuming that the plasma contains electrons and only one species of ions (with $Z = 1$) with bi-maxwellian equilibrium distribution[17] functions

$$F_r^{(0)} = \frac{1}{(2\pi)^{3/2}} \frac{m_r^{3/2}}{T_{\perp r}^{(0)} T_{\| r}^{(0)1/2}} \exp \Big\{ - \Big(\frac{m_r}{2T_{\| r}^{(0)}} v_\|^2 + \frac{m_r}{2T_{\perp r}^{(0)}} v_\perp^2 \Big) \Big\}. \tag{36}$$

Using the quasi-neutrality conditions that prescribe $n_r = n^{(0)}$ and $\rho_r^{(1)} = m_r n^{(1)}$, one obtains

$$\mathcal{L}_r = -n^{(0)} q_r^2 \frac{1}{T_{\| r}^{(0)}} \mathcal{W}_r \ , \ \mathcal{M}_r = -n^{(0)} q_r \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}} \mathcal{W}_r \ , \ \mathcal{N}_r = -2n^{(0)} \frac{T_{\perp r}^{(0)2}}{T_{\| r}^{(0)}} \mathcal{W}_r \ , \tag{37}$$

$$\mathcal{O}_r = -n^{(0)} m_r \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}} \mathcal{W}_r \ , \ \mathcal{P}_r = -n^{(0)} q_r \frac{1}{T_{\| r}^{(0)}} \mathcal{W}_r \tag{38}$$

where, normalizing the propagation velocity of the wave by the the thermal velocity $v_{th,r} = \sqrt{T_{\| r}^{(0)}/m_r}$ in the form $c_r = \lambda / v_{th,r}$, one writes

$$\mathcal{W}_r \equiv \mathcal{W}(c_r) = \frac{1}{\sqrt{2\pi}} \mathrm{P} \int \frac{\zeta e^{-\zeta^2/2}}{\zeta - c_r} d\zeta + \sqrt{\frac{\pi}{2}} c_r e^{-c_r^2/2} \mathcal{H}_\xi, \tag{39}$$

or[16]

$$\mathcal{W}(c_r) = 1 - c_r e^{-\frac{c_r^2}{2}} \int e^{\frac{\zeta^2}{2}} d\zeta + \sqrt{\frac{\pi}{2}} c_r e^{-c_r^2/2} \mathcal{H}_\xi. \tag{40}$$

This function is related to the plasma response function $\mathcal{R}$ used by SHD by $\mathcal{W}(X) = \mathcal{R}(X/\sqrt{2})$.

One has

$$\frac{T^{(1)}_{\|r}}{T^{(0)}_{\|r}} \equiv \frac{p^{(1)}_{\|r}}{p^{(0)}_{\|r}} - \frac{n^{(1)}}{n^{(0)}} = (c_r^2 - 1 + \mathcal{W}_r^{-1})\Big(\frac{n^{(1)}}{n^{(0)}} - A\Big) \tag{41}$$

$$\frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} \equiv \frac{p^{(1)}_{\perp r}}{p^{(0)}_{\perp r}} - \frac{n^{(1)}}{n^{(0)}} = \Big(1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}} \mathcal{W}_r\Big) A \tag{42}$$

and

$$\frac{q^{(1)}_{\|r}}{v_{th,r} p^{(0)}_{\|r}} = c_r (c_r^2 - 3 + \mathcal{W}_r^{-1})\Big(\frac{n^{(1)}}{n^{(0)}} - A\Big) \tag{43}$$

$$\frac{q^{(1)}_{\perp r}}{v_{th,r} p^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}} c_r \mathcal{W}_r A, \tag{44}$$

or

$$\frac{q^{(1)}_{\|r}}{v_{th,r} p^{(0)}_{\|r}} = \frac{c_r (c_r^2 - 3 + \mathcal{W}_r^{-1})}{c_r^2 - 1 + \mathcal{W}_r^{-1}} \frac{T^{(1)}_{\|r}}{T^{(0)}_{\|r}} \tag{45}$$

$$\frac{q^{(1)}_{\perp r}}{v_{th,r} p^{(0)}_{\perp r}} = -\frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}} \frac{c_r \mathcal{W}_r}{1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\|r}} \mathcal{W}_r} \frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}}. \tag{46}$$

The formulae for the pressures are consistent with those of Quataert, Dorland and Hammett[13] where the parameter c_r is defined for monochromatic perturbations as $c_r = \frac{\omega}{k} \frac{1}{v_{th,r}}$. They also permit one to calculate generalized polytropic indices.[14]

In the nearly isothermal limit ($c_r \ll 1$), $\mathcal{W}_r \approx 1 - c_r^2 + \sqrt{\frac{\pi}{2}} c_r \mathcal{H}_\xi$ and one gets $q^{(1)}_{\|r} = -\sqrt{\frac{8}{\pi}} v_{th,r} n^{(0)} \mathcal{H}_\xi T^{(1)}_{\|r}$ independent of c_r and $q^{(1)}_{\perp r} \ll 1$. Differently, in the adiabatic limit ($c_r \gg 1$), $\mathcal{W}_r \approx -1/c_r^2 - 3/c_r^4$ and the heat fluxes are negligible.

IV. TOWARDS A LANDAU FLUID CLOSURE

A. Heat flux closures

The expressions for the heat fluxes obtained in Section 3, that involve Alfvén wave velocity through the parameter c_r, are specific of one-directional long Alfvén waves propagating along an ambient magnetic field. Some heuristic transformations are necessary in order to extend the closure formulae to more general situations. It is thus appropriate at this step to come back to the original variables. In order to derive a closure in a context where all types of waves coexist, the parameter c_r should not be interpreted as the ratio of the (signed) Alfvén speed to the thermal velocity of species r, but should more generally be viewed as the ratio $-\frac{1}{v_{th,r}} \partial_t \partial_x^{-1}$. Furthermore, eqs. (45)-(46) for the heat fluxes, which involve complicated (and even non-analytic) functions of c_r, cannot directly be used to close the fluid hierarchy. Simple formulae can nevertheless be obtained if one prescribes a linear or homographic dependence on c_r. Noting that the plasma response function has the form $\mathcal{W}(c_r) = A(|c_r|) + c_r B(|c_r|)\mathcal{H}$, where the functions A and B do not depend on the sign of c_r, we propose to take A and B constant. Here $\mathcal{H}$ is the Hilbert transform with respect to x. Plugging this ansatz in eqs. (45)-(46), the parallel and perpendicular heat fluxes take the form

$$\frac{q^{(1)}_{\|r}}{v_{th,r} p^{(0)}_{\|r}} = \mathcal{F}_\| \Big(-\frac{1}{v_{th,r}} \partial_t \partial_x^{-1}\Big) \frac{T^{(1)}_{\|r}}{T^{(0)}_{\|r}} \tag{47}$$

$$\frac{q^{(1)}_{\perp r}}{v_{th,r} p^{(0)}_{\perp r}} \quad = \mathcal{F}^1_\perp\left(-\frac{1}{v_{th,r}}\partial_t \partial_x^{-1}\right)\frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} + \mathcal{F}^2_\perp\left(-\frac{1}{v_{th,r}}\partial_t \partial_x^{-1}\right) A \tag{48}$$

where in the homographic approximation

$$\mathcal{F}_\parallel(X) = (Q^3_\parallel + Q^4_\parallel X \mathcal{H})^{-1}(Q^1_\parallel X + Q^2_\parallel \mathcal{H}) \tag{49}$$

$$\mathcal{F}^1_\perp(X) = (Q^3_\perp + Q^4_\perp X \mathcal{H})^{-1}(Q^1_\perp X + Q^2_\perp \mathcal{H}) \tag{50}$$

$$\mathcal{F}^2_\perp(X) = Q^5_\perp X + Q^6_\perp \mathcal{H}. \tag{51}$$

The linear fit corresponds to taking $Q^3_\parallel = Q^3_\perp = 1$ and $Q^4_\parallel = Q^4_\perp = 0$.

The ansatz for the parallel heat flux results directly from eq. (45). The expression for the perpendicular heat flux is suggested by both eqs. (44) and (46) and the constraint that $q^{(1)}_{\perp r}$ should vanish in the limit $c_r \to 0$. This functional form is however not unique and using the same number of free parameters, an alternative form is for example

$$\mathcal{F}^1_\perp(X) = (Q'^3_\perp + Q'^4_\perp X \mathcal{H})^{-1}(Q'^1_\perp X + Q'^2_\perp \mathcal{H}) \tag{52}$$

$$\mathcal{F}^2_\perp(X) = (Q'^3_\perp + Q'^4_\perp X \mathcal{H})^{-1}(Q'^5_\perp X + Q'^6_\perp \mathcal{H}). \tag{53}$$

The coefficients $Q^i_\parallel$ and $Q^i_\perp$ (or $Q'^i_\perp$) are chosen in a way that ensures the correct asymptotic behavior of the heat fluxes in both the isothermal ($c_r \ll 1$) and adiabatic ($c_r \gg 1$) limits. For the parallel heat flux, the fit is made between eqs. (47) and (45). For the perpendicular heat flux, it is first necessary to express temperature fluctuations in terms of magnetic perturbations in eq. (48) by means of eq. (42). The fit is then performed by comparison with eq. (44).

This procedure results in approximating the response function of the plasma in eq. (45) by

$$\mathcal{W}_\parallel(X) = \frac{\mathcal{F}_\parallel(X) - X}{(1 - X^2)\mathcal{F}_\parallel(X) + X(X^2 - 3)} \tag{54}$$

and in eq. (46) by

$$\mathcal{W}_\perp(X) = \frac{T^{(0)}_{\parallel r}}{T^{(0)}_{\perp r}} \frac{\mathcal{F}^1_\perp(X) + \mathcal{F}^2_\perp(X)}{\mathcal{F}^1_\perp(X) - X}. \tag{55}$$

In the case where $\mathcal{F}_\parallel$ is taken linear in X, one obtains $Q^1_\parallel = 0$ and $Q^2_\parallel = -\sqrt{\frac{8}{\pi}}$, leading to the isothermal limit mentioned in Section III.C

$$\frac{q^{(1)}_{\parallel r}}{v_{th,r} p^{(0)}_{\parallel r}} = -\sqrt{\frac{8}{\pi}} \mathcal{H} \frac{T^{(1)}_{\parallel r}}{T^{(0)}_{\parallel r}}. \tag{56}$$

In this case $\mathcal{W}_\parallel$ reduces to the three-pole model $\mathcal{W}_3$ of $\mathcal{W}$, as given by eq. (43) of SHD.

When $\mathcal{F}_\parallel$ is taken homographic in X, one has $Q^1_\parallel = 0$, $Q^2_\parallel = -\sqrt{\frac{8}{\pi}}$, $Q^3_\parallel = 1$, $Q^4_\parallel = -\sqrt{\frac{8}{\pi}}(\frac{3\pi}{8} - 1)$ which leads for $\mathcal{W}_\parallel$, to the four-pole approximation $\mathcal{W}_4$. The parallel heat flux is now determined by the partial differential equation

$$\left(\frac{d}{dt} + \frac{v_{th,r}}{\sqrt{\frac{8}{\pi}}(1 - \frac{3\pi}{8})}\mathcal{H}\partial_x\right) \frac{q^{(1)}_{\parallel r}}{v_{th,r} p^{(0)}_{\parallel}} = \frac{1}{1 - \frac{3\pi}{8}} v_{th,r} \partial_x \frac{T^{(1)}_{\parallel r}}{T^{(0)}_{\parallel r}} \tag{57}$$

where, to restore Galilean invariance, convective derivatives have been substituted to partial time derivatives. In the isothermal limit where the time derivative is neglected, eq. (57) reduces to eq. (56).

Concerning the transverse heat flux $q^{(1)}_{\perp r}$, the linear form of $\mathcal{F}^1_\perp$ leads to $Q^1_\perp = Q^4_\perp = Q^5_\perp = 0$, $Q^2_\perp = -\sqrt{\frac{2}{\pi}}$ and $Q^6_\perp = \sqrt{\frac{2}{\pi}}\left(1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}}\right)$. In this case the approximate response function $\mathcal{W}_\perp$ reduces to the one-pole model $\mathcal{W}_1$ of $\mathcal{W}$. The closure for the perpendicular heat flux then reads

$$\frac{q^{(1)}_{\perp r}}{v_{th,r} p^{(0)}_{\perp r}} = -\sqrt{\frac{2}{\pi}} \mathcal{H} \frac{T^{(1)}_{\perp r}}{T^{(0)}_{\perp r}} + \sqrt{\frac{2}{\pi}}\left(1 - \frac{T^{(0)}_{\perp r}}{T^{(0)}_{\parallel r}}\right)\mathcal{H} A, \tag{58}$$

that reproduces eq. (40) of SHD. In the case where $\mathcal{F}_\perp^1$ is homographic in X, one has $Q_\perp^1 = 1 + \sqrt{\frac{\pi}{2}} f$, $Q_\perp^2 = Q_\perp^4 = f$, $Q_\perp^3 = 1$, $Q_\perp^5 = 0$, $Q_\perp^6 = (\frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}} - 1) f$, with $f = -\sqrt{\frac{8}{\pi}} \left(1 + \sqrt{1 + \frac{8}{\pi}(\frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}} - 1)}\right)^{-1}$. When f is complex of the form $f_1 + i f_2$, it should be understood as $f_1 + f_2 \mathcal{H}$. It follows that $q_\perp^{(1)}$ obeys

$$\left(\frac{d}{dt} + \frac{v_{th,r}}{f} \mathcal{H} \partial_x\right) \frac{q_{\perp r}^{(1)}}{v_{th,r} p_{\perp r}^{(0)}} = \left(\frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}} - 1\right) f \frac{d}{dt} \mathcal{H} \left(A - \frac{T_{\perp r}^{(1)}}{T_{\perp r}^{(0)}}\right) + v_{th,r} \partial_x \left(\left(1 - \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}}\right) A - \frac{T_{\perp r}^{(1)}}{T_{\perp r}^{(0)}}\right). \tag{59}$$

A simpler equation is obtained when using the ansätze (52)-(53). One has $Q_\perp^{'1} = Q_\perp^{'5} = 0$, $Q_\perp^{'2} = Q_\perp^{'4} = -\sqrt{\frac{2}{\pi}}$ and $Q_\perp^{'6} = \sqrt{\frac{2}{\pi}} (1 - \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}})$. Equation (59) is then replaced by

$$\left(\frac{d}{dt} - \sqrt{\frac{\pi}{2}} v_{th,r} \mathcal{H} \partial_x\right) \frac{q_{\perp r}^{(1)}}{v_{th,r} p_{\perp r}^{(0)}} = v_{th,r} \partial_x \left(\left(1 - \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}}\right) A - \frac{T_{\perp r}^{(1)}}{T_{\perp r}^{(0)}}\right) \tag{60}$$

which turns out to be much simpler.

In both approaches the approximate response function $\mathcal{W}_\perp$ reduces to the two-pole model $\mathcal{W}_2$ of $\mathcal{W}$. Moreover, the isothermal limit of eq. (60) reduces to the perpendicular heat flux given by eq. (58).

The refined "3+1" Landau closure (57) and (60) reproduce the same response functions as the "4+2" closure of SHD. These equations identify with the linearized heat flux equations (32)-(35) of SHD when prescribing bi-Maxwellian values for the fourth-order moments.

B. The Hall Landau-fluid model

The quasi-neutrality condition implying a simple relation between the velocities of the ions and the electrons, it is convenient to consider the density $\rho = \sum_r \rho_r = n \sum_r m_r$ and the plasma velocity $u = \frac{1}{\rho} \sum_r \rho_r u_r$. Considering as above a unique ion species with $Z = 1$, one has at dominant order $u = u_i = u_e$[20] and thus

$$\partial_t \rho + \nabla \cdot (u \rho) = 0 \tag{61}$$

$$\rho \left(\partial_t u + u \cdot \nabla u\right) + \nabla \cdot (P^{(0)} + \Pi) - \frac{1}{c} j \times b = 0, \tag{62}$$

where $j = \dfrac{c}{4\pi} \nabla \times b$ is the current vector, $P^{(0)} = \sum_r P_r^{(0)}$ and where, to leading order in Ω_i^{-1}, the finite Lamor radius corrections (FLR) Π are given by[5]

$$\Pi_{yy} = -\Pi_{zz} = -\frac{p_{\perp i}}{2\Omega_i} (\partial_y u_z + \partial_z u_y) \tag{63}$$

$$\Pi_{xx} = 0 \tag{64}$$

$$\Pi_{yz} = \Pi_{zy} = \frac{p_{\perp i}}{2\Omega_i} (\partial_y u_y - \partial_z u_z) \tag{65}$$

$$\Pi_{zx} = \Pi_{xz} = -\frac{1}{\Omega_i} [p_{\perp i} (\partial_x u_y - \partial_y u_x) - 2 p_{\| i} \partial_x u_y] \tag{66}$$

$$\Pi_{xy} = \Pi_{yx} = \frac{1}{\Omega_i} [p_{\perp i} (\partial_x u_z - \partial_z u_x) - 2 p_{\| i} \partial_x u_z]. \tag{67}$$

Such a simple description of the FLR corrections are sufficient when dealing with parallel propagating waves. Corrective terms have nevertheless to be retained in cases such as obliquely propagating Alfvén waves for which the dispersion coefficient scales like Ω_i^{-2}.[18,19]

Furthermore, the parallel and perpendicular components of the gyrotropic pressure of each particle species obey

$$\partial_t p_{\| r} + \nabla \cdot (u\, p_{\| r}) + \nabla \cdot (\widehat{b}\, q_{\| r}) + 2 p_{\| r}\, \widehat{b} \cdot \nabla u \cdot \widehat{b} - 2 q_{\perp r} \nabla \cdot \widehat{b} = 0, \tag{68}$$

$$\partial_t p_{\perp r} + \nabla \cdot (u\, p_{\perp r}) + \nabla \cdot (\widehat{b}\, q_{\perp r}) + p_{\perp r} \nabla \cdot u - p_{\perp r}\, \widehat{b} \cdot \nabla u \cdot \widehat{b} + q_{\perp r} \nabla \cdot \widehat{b} = 0. \tag{69}$$

The induction equation including the Hall-effect is

$$\partial_t b - \nabla \times (u \times b) = -\frac{m_i c}{q_i} \nabla \times \Big[\frac{1}{4\pi\rho}(\nabla \times b) \times b - \frac{1}{\rho}\nabla \cdot P_e^{(0)}\Big]. \tag{70}$$

This system is closed by prescribing that the heat fluxes evolve according to

$$\Big(\frac{d}{dt} + \frac{v_{th,r}}{\sqrt{\frac{8}{\pi}}(1-\frac{3\pi}{8})}\mathcal{H}\partial_x\Big)\frac{q_{\| r}}{v_{th,r}p_{\|}^{(0)}} = \frac{1}{1-\frac{3\pi}{8}} v_{th,r}\partial_x \frac{T_{\| r}}{T_{\| r}^{(0)}} \tag{71}$$

$$\left(\frac{d}{dt} - \sqrt{\frac{\pi}{2}} v_{th,r}\mathcal{H}\partial_x\right)\frac{q_{\perp r}}{v_{th,r}p_{\perp r}^{(0)}} = v_{th,r}\partial_x\left(\Big(1 - \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}}\Big)\frac{|b|}{B_0} - \frac{T_{\perp r}}{T_{\perp r}^{(0)}}\right), \tag{72}$$

the temperatures being given by $p_{\| r} = nT_{\| r}$ and $p_{\perp r} = nT_{\perp r}$.

This formulation where Landau damping is computed by integration along the ambient field is restricted to weakly nonlinear dynamics such as for example, that described by long Alfvén wave asymptotics. The extension of this model to more nonlinear regimes would require integration along perturbed field lines[12] and also the inclusion of stronger parallel nonlinearities as mentioned by SHD. Furthermore, the above formulation assumes that the Hall term does not arise at the level of the closure equations. This prescription, that is justified for parallel Alfvén waves and also for oblique magnetosonic waves, can become questionable in the case of oblique Alfvén waves[19]

V. LANDAU-FLUID DESCRIPTION OF LONG DISPERSIVE ALFVÉN WAVES

The long wave asymptotics performed by Mjølhus and Wyller[4] on eqs. (61) and (62), (63)-(67) and (70) leads to eq. (9) without mean fields, where in $\widetilde{P} = \frac{B_0^2}{4\pi}\widetilde{A} + \widetilde{p}_\perp^{(1)}$ the transverse pressure $\widetilde{p}_\perp^{(1)}$ is determined by these authors using a kinetic approach based on the guiding center equation. Note that the dispersion term in eq. (9) originates from both the Hall term and the electron pressure gradient included in the induction equation and from finite Larmor radius corrections to the gyrotropic pressure. An alternative approach consisting in a long wave asymptotics performed directly on eqs. (1)-(3) is used by Verheest.[11]

In this section we show that a reductive perturbation expansion on the above Hall Landau-fluid model is able to accurately reproduce the full KDNLS system (9), (10) and (11) derived from the Vlasov-Maxwell equations, with no reference to the kinetic theory. At this level one only needs to calculate the fluctuating transverse pressure and the equations for the mean fields.

By eliminating u using eq. (61) and (70),[9] it is convenient to rewrite eqs. (68)-(69) in the form of CGL (for Chew-Goldberger-Low) equations[8] including heat fluxes[13] together with Hall effect and electron pressure gradient.[15] They read

$$\rho|b|\frac{d}{dt}\left(\frac{p_{\perp r}}{\rho|b|}\right) = -\nabla\cdot(q_{\perp r}\widehat{b}) - q_{\perp r}\nabla\cdot\widehat{b} + p_{\perp r} b\cdot[\nabla \times \frac{m_i c}{\rho q_i}(j\times b - \nabla P_e^{(0)})] \tag{73}$$

$$\frac{\rho^3}{|b|^2}\frac{d}{dt}\left(\frac{p_{\| r}|b|^2}{\rho^3}\right) = -\nabla\cdot(q_{\| r}\widehat{b}) + 2q_{\perp r}\nabla\cdot\widehat{b} - 2p_{\| r} b\cdot\Big[\nabla\times\frac{m_i c}{\rho q_i}(j\times b - \nabla\cdot P_e^{(0)})\Big]. \tag{74}$$

When plugging the rescaled variables in these equations, one notices that the Hall and electron pressure corrections are negligible in the present asymptotics. When considering the fluctuating contributions, the time derivative is, to leading order, replaced by $-\lambda\partial_\xi$, the divergence $\nabla\cdot\widehat{b}$ reduces to $-\epsilon^4\partial_\xi A$ and the right-hand-side of eq. (73) reduces to leading order to $-\widehat{b}\cdot\nabla \approx -\epsilon^2\partial_x i\widetilde{q}_\perp$. This reproduces eq. (25) for the fluctuating transverse heat flux $\widetilde{q}_\perp^{(1)}$. A similar argument applied to eq. (74) for the parallel pressure reproduces eq. (26) for $\widetilde{q}_\|^{(1)}$.

From expansion to leading order of eq. (71), it is easy to see, by carrying backwards the closure procedure, that the parallel heat flux fluctuation $\widetilde{q}_\|^{(1)}$ predicted by the Hall Landau-fluid model is given by eq. (45) where $\mathcal{W}$ should be replaced by $\mathcal{W}_4$. Similarly the corresponding perpendicular heat flux fluctuation $\widetilde{q}_\perp^{(1)}$ is given by eq. (46) with $\mathcal{W}$ replaced by $\mathcal{W}_2$.

Plugging these heat fluxes in eqs. (25) and (26) one gets

$$\widetilde{p}_\perp^{(1)} = p_\perp^{(0)}\frac{\widetilde{n}^{(1)}}{n^{(0)}} + \sum_r p_{\perp r}^{(0)}\Big(1 - \frac{T_{\perp r}^{(0)}}{T_{\| r}^{(0)}}\mathcal{W}_2(c_r)\Big)\widetilde{A} \tag{75}$$

$$\widetilde{p}_{\|}^{(1)} = \sum_r p_{\|r}^{(0)}\Big(c_r^2 + \mathcal{W}_4^{-1}(c_r)\Big)\frac{\widetilde{n}^{(1)}}{n^{(0)}} - \sum_r p_{\|r}^{(0)}\Big(c_r^2 - 1 + \mathcal{W}_4^{-1}(c_r)\Big)\widetilde{A}. \tag{76}$$

Furthermore, the mass, longitudinal velocity and induction equations lead to

$$\frac{\widetilde{u}_{\|}^{(1)}}{\lambda} + \frac{\widetilde{b}_x^{(1)}}{B_0} - \frac{\widetilde{n}^{(1)}}{n^{(0)}} = 0 \tag{77}$$

$$-\lambda\rho^{(0)}\widetilde{u}_{\|}^{(1)} + \lambda^2\rho^{(0)}\frac{\widetilde{|b|^2}}{2B_0^2} + \widetilde{p}_{\|}^{(1)} + (p_\perp^{(0)} - p_{\|}^{(0)})\widetilde{A} = 0. \tag{78}$$

Equations (76), (77) and (78) yield

$$\frac{\widetilde{n}^{(1)}}{n^{(0)}} = \Big(1 - \frac{p_\perp^{(0)}}{\sum_r p_{\|r}^{(0)}(c_r^2 + \mathcal{W}_4^{-1}(c_r)) - \lambda^2\rho^{(0)}}\Big)\widetilde{A} \tag{79}$$

Substituting in eq. (75) and noting that $\lambda^2\rho^{(0)} = \sum_r c_r^2 p_{\|r}^{(0)}$, one gets

$$\widetilde{p}_\perp^{(1)} = 2p_\perp^{(0)}\widetilde{A} - \Big(\frac{p_\perp^{(0)2}}{\sum_r p_{\|r}^{(0)}\mathcal{W}_4^{-1}(c_r)} + \sum_r p_{\perp r}^{(0)}\frac{T_{\perp r}^{(0)}}{T_{\|r}^{(0)}}\mathcal{W}_2(c_r)\Big)\widetilde{A} \tag{80}$$

Defining the operators $\mathcal{L}_s$, $\mathcal{M}_s$, $\mathcal{N}_s$ with $s = 2$ or 4 by means of eq. (37) up to the replacement of $\mathcal{W}$ by the corresponding two or three-pole approximation, one gets after some straightforward algebra

$$\frac{p_\perp^{(0)2}}{\sum_r p_{\|r}^{(0)}\mathcal{W}_4^{-1}(c_r)} = \mathcal{M}_4^2\mathcal{L}_4^{-1} - \frac{\mathcal{N}_4}{2} \tag{81}$$

and thus

$$\widetilde{p}_\perp^{(1)} = \Big(2p_\perp^{(0)} + \frac{\mathcal{N}_2 + \mathcal{N}_4}{2} - \mathcal{M}_4^2\mathcal{L}_4^{-1}\Big)\widetilde{A}. \tag{82}$$

This result provides a substitute for the kinetic computation of the perpendicular pressure fluctuations based on the guiding center[4] or the full Vlasov-Maxwell equations.[1,2] The good accuracy of this approximation is visible in Figs. 1 and 2 that display the contributions p^R and p^I to the transverse pressure $\frac{\widetilde{p}_\perp^{(1)}}{p_\perp^{(0)}} = -(p^R + p^I\mathcal{H})\widetilde{A}$ versus $\beta^{-1/2}$, where $\beta = T_{\|e}^{(0)}/(m_i\lambda^2)$, for various conditions on the temperatures. When all the temperatures are equal (Figs. 1a and 1b) the curves presented in figures 3 and 4 of SHD are reproduced. Figure 2 reproduces the quasi-singularity that develops near $\beta = 1$ in the regime where the electron temperature strongly exceeds that of the ions. Figure 3 illustrates the influence of an ion temperature anisotropy. In this case, the curve is plotted as a function of $\beta_{\|p}^{-1/2}$ since β is bounded from above.

We now turn to the determination of the longitudinally averaged quantities. It is convenient to sum eqs. (73)-(74) over the species before expanding at order ϵ^6 (after expliciting the convective time derivative) and averaging over the ξ variable. It is noticeable that only the fluctuating part of the heat fluxes, given by eqs. (25)-(26) after summation over the species, enter these equations since $\langle b_\perp\rangle = 0$ and $\nabla\cdot\widehat{b} = -\partial_\xi A$. After some algebra one gets

$$\partial_\tau\Big(\frac{\langle p_\perp^{(1)}\rangle}{p_\perp^{(0)}} - \frac{\langle\rho^{(1)}\rangle}{\rho^{(0)}} - \langle A\rangle\Big) = 0 \tag{83}$$

$$\partial_\tau\Big(\frac{\langle p_{\|}^{(1)}\rangle}{p_{\|}^{(0)}} - 3\frac{\langle\rho^{(1)}\rangle_\xi}{\rho^{(0)}} + 2\langle A\rangle\Big) = -\frac{2\lambda}{p_{\|}^{(0)}}\langle\widetilde{p}_\perp^{(1)}\partial_\xi\widetilde{A}\rangle. \tag{84}$$

This reproduces exactly eqs. (19) and (20) after substitution of the exact perpendicular fluctuating pressure (22) which, when starting with the (unclosed) fluid hierarchy, turns out to be the only quantity to be computed in the long-wave aymptotics by means of the kinetic theory. In the context of the Hall Landau-fluid model, this quantity is approximated by eq. (82).

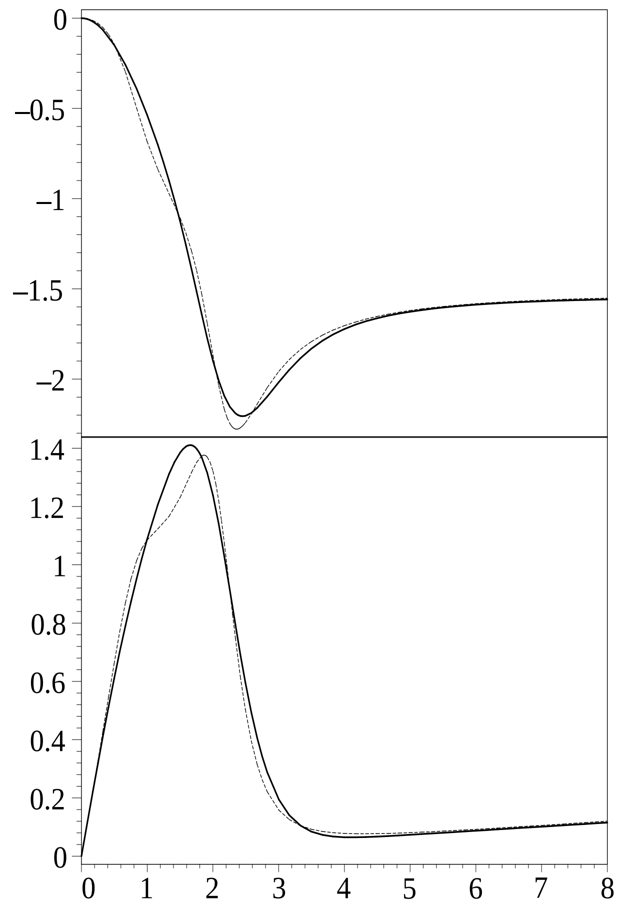

FIG. 1: Contributions p^R (top) and p^I (bottom) of the perpendicular pressure response versus $\beta^{-1/2}$, where $\beta = T_{\| e}/(m_i\lambda^2)$, for $T_{\| e} = T_{\| i} = T_{\perp e} = T_{\perp i}$. The solid line refers to the exact (long-wave) kinetic calculation and the thin dashed line to the Landau-fluid closure. The plasma is constituted of protons and electrons.

From the mass and longitudinal induction equation averaged along the ξ coordinate, one easily gets after using (77)

$$\partial_\tau \frac{\langle \rho^{(1)} \rangle}{\rho^{(0)}} = \partial_\tau \frac{\langle b_x^{(1)} \rangle}{B_0}. \tag{85}$$

Writing the longitudinal velocity equation at order ϵ^6 and averaging over the ξ variable, one gets

$$\begin{aligned} \partial_\tau \langle u_\| \rangle = \nabla_\perp \cdot \Big[& \frac{\lambda}{B_0} \langle \widetilde{u}_\|^{(1)} b_\perp^{(0)} \rangle + \frac{1}{4\pi\rho^{(0)}} \langle \widetilde{b}_\|^{(1)} b_\perp^{(0)} \rangle \\ & + \frac{p_\|^{(0)} - p_\perp^{(0)}}{\rho^{(0)}} \Big(\frac{\langle b_\|^{(1)} b_\perp^{(0)} \rangle}{B_0^2} + \frac{\langle |b_\perp^{(0)}|^2 b_\perp^{(0)} \rangle}{B_0^3} \Big) - \frac{1}{\rho^{(0)} B_0} \langle (p_\|^{(1)} - p_\perp^{(1)}) b_\perp^{(0)} \rangle \Big]. \end{aligned} \tag{86}$$

Note that the FLR terms do not contribute. Using (78), one recovers eq. (16).

Equation (17) is obtained when deriving the KDNLS equation and prescribing that a mean transverse magnetic field is not driven. Finally, proceeding as in paper I, eq. (18) is readily recovered from the already established equations as a consequence of mass conservation.

We thus conclude that the Hall-Landau fluid model reproduces exactly the long-wave asymptotics performed on the Vlasov-Maxwell equation, up to the substitution of the fluctuating perpendicular pressure $\widetilde{p}_\perp^{(1)}$ by the approximate value given by eq. (82).

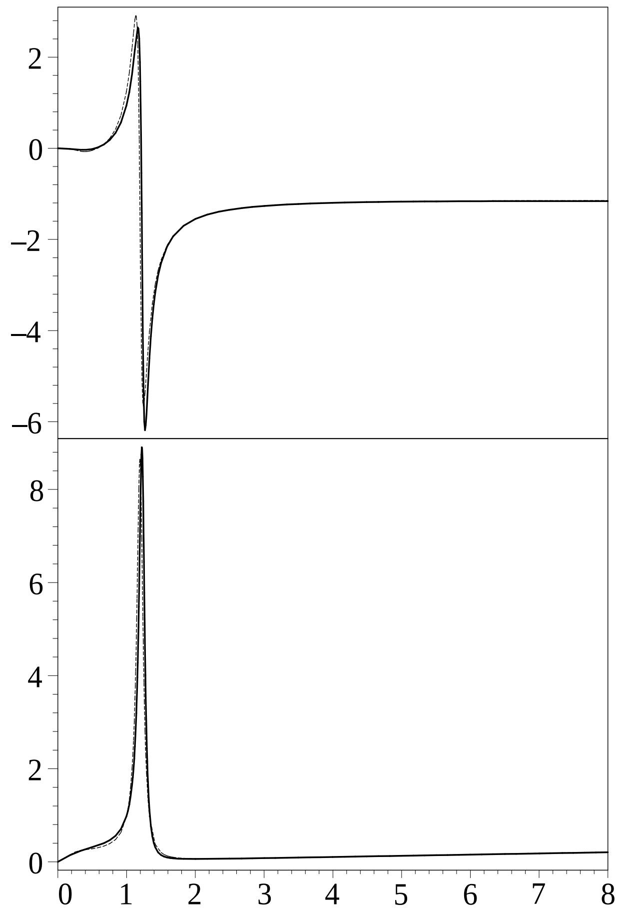

FIG. 2: Same as Fig. 1 for isotropic ion and electron temperatures with $T_e = 8T_i$.

VI. CONCLUSION

In this paper we have derived a closed system of equations that can be viewed as a Hall Landau-fluid model, suitable to describe magnetohydrodynamic waves in a collisionless plasma where the Landau damping is the main dissipation process. It is derived for a plasma of electrons and one species of $Z = 1$ ions, with bi-Maxwellian equilibrium distributions. This model appears as a refined formulation of SHD's 3+1 Landau closure, that reproduces the same response functions as the 4+2 closure without involving the full dynamical equations governing the heat fluxes. Furthermore, the Hall-effect retained in the generalized Ohm's law makes the Alfvén waves dispersive, permitting in particular modulational instabilities and wave collapse. Is is demonstrated that this Hall-Landau fluid model accurately describes the long-wave dynamics of parallel dispersive Alfvén waves. The KDNLS equation derived in paper I from the full Vlasov-Maxwell equations is exactly reproduced from a long-wave asymptotics performed on the Hall Landau-fluid model up to an approximation of the plasma response function. This fluid model should provide an efficient tool to simulate regimes of dispersive Alfvén wave turbulence and estimate the resulting heating of the plasma. A question concerns the numerical stability of this Hall Landau-fluid model. It is unclear whether a numerical viscosity is needed in the nonlinear dynamics to prevent formation of arbitrary small scales.

Acknowledgments

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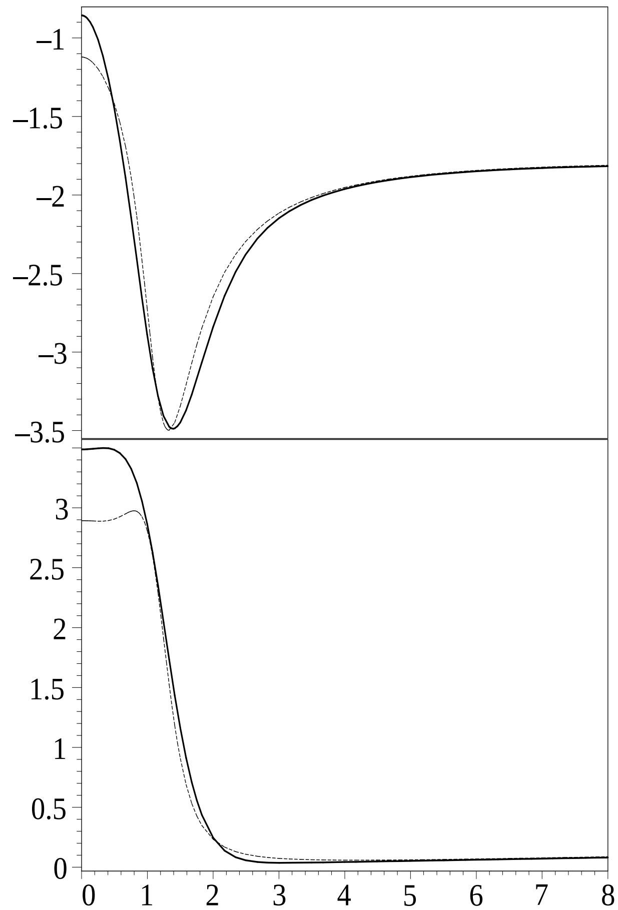

FIG. 3: Contributions p^R (top) and p^I (bottom) of the perpendicular pressure response versus $\beta_{\|p}^{-1/2}$, for $T_{\perp e} = T_{\| e} = T_{\| i}$ and $T_{\perp i} = 3T_{\| i}$.

[1] T. Passot and P.L. Sulem, "A long-wave model for Alfvén wave trains in a collisionless plasma: I. Kinetic theory", Phys. Plasmas, this issue.
[2] A. Rogister, Phys. Fluids 12, 2733 (1971).
[3] M.V Medvedev and P.H. Diamond, P.H, Phys. Plasmas 3, 863 (1996).
[4] E. Mjølhus, and J. Wyller, *J. Plasma Phys.* **40**, 299-318 (1988).
[5] N. Yajima, Prog. Theor. Phys. 36, 1 (1966).
[6] G.W. Hammett and W. Perkins, Phys. Rev. Lett. **64**, 3019 (1990).
[7] P.B. Snyder, G.W. Hammett, and W. Dorland, Phys. Plasmas 4, 3974 (1997).
[8] G.F. Chew, M.L. Goldberger, and F.E. Low, Proc. Roy. Soc. (London) A 236, 112 (1956).
[9] R.M. Kulsrud in *Handbook of Plasma Physics*, edited by M.N. Rosenbluth and R.Z. Sagdeev, Vol. 1 *Basic Plasma Physics* edited by A.A. Galeev and R.N. Sudan, p. 115, (North Holland, New York 1983).
[10] F. Verheest and B. Buti, J. Plasma Physics 47, 15 (1992).
[11] F. Verheest, J. Plasma Physics 47, 25 (1992).
[12] J. Finn and R. Gerwin, Phys. Plasmas 3, 2469 (1996).
[13] E. Quataert, W. Dorland, and G.W. Hammett, Astrophys. J. 577, 524 (2002).
[14] G. Belmont and C. Mazelle, J. Geophys. Res. 97, 8327 (1992).
[15] Khanna, M. and Rajaram, R., J. Plasma Phys. 28, 459 (1982).
[16] S. Ischimaru, *Basic Principles of Plasma Physics: A Statistical Approach*, Frontiers in Physics, Benjamin (1973).
[17] T. Flå, E. Mjølhus, and J. Wyller, Physica Scripta 40, 219 (1989).
[18] V.A. Marchenko, R.E. Denton, and M.K. Hudson, Phys. Plasmas 3, 3861 (1996).
[19] T. Passot and P.L. Sulem, "A fluid description for Landau damping of dispersive MHD waves", submitted to Nonlinear Proc. Geophys.
[20] This equality is not required in the usual single-fluid MHD approximation where the total pressure, instead of being viewed

as the sum of the partial pressures, is defined in terms of the deviations from the barycentric plasma velocity. It is however here necessary in order to write individual pressure equations for the ions and electrons. Moreover, in the present ordering, this assumption is not inconsistent with retaining the Hall term in the induction equation.